Fall 2015 Welcome

Steven F. Bellenot

Department of Mathematics
Florida State University

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There are infinitely many primes

- Northshield 2015 a one liner
- Furstenberg 1955 (at age 20) a topological proof
- Euler 1737 (age 30) $\sum 1/p = \infty$

Talking Points

Grade Distributions
Email
Accommodations
ALEKS

ALEKS

"First Time in College" students in mac1114, mac1140, mac2233, mac2311 and mac2312 are required to take aleks.
And they must use the FSU Summer/Fall 2015 cohort.
Yes even if they have credit for previous class.
Yes even if they took mac1105 this summer.
Yes even if their advisor told them otherwise.
Yes even if they took ALEKS for someone else.

ALEKS is not

NOT a way to jump from MAC1105 to MAC2311
NOT a way to avoid repeating a course
NOT a way to avoid trigonometry – separate trig score

Let $P = \prod_p p$ and suppose there are only finitely many primes so P is a finite integer.

$$0 < \prod_p \sin\left(\frac{\pi}{p}\right) = \prod_p \sin\left(\pi \frac{1+2P}{p}\right) = 0$$

Email

Advisors (other than Pamela or Kari) are not your friend

- Do not reply to email from students wanting to add your class, just forward them to advisor@math.fsu.edu

Topology

- the union of open sets is open
- the complement of an open set is closed and the complement of a closed set is open
- a finite union of closed sets is closed.

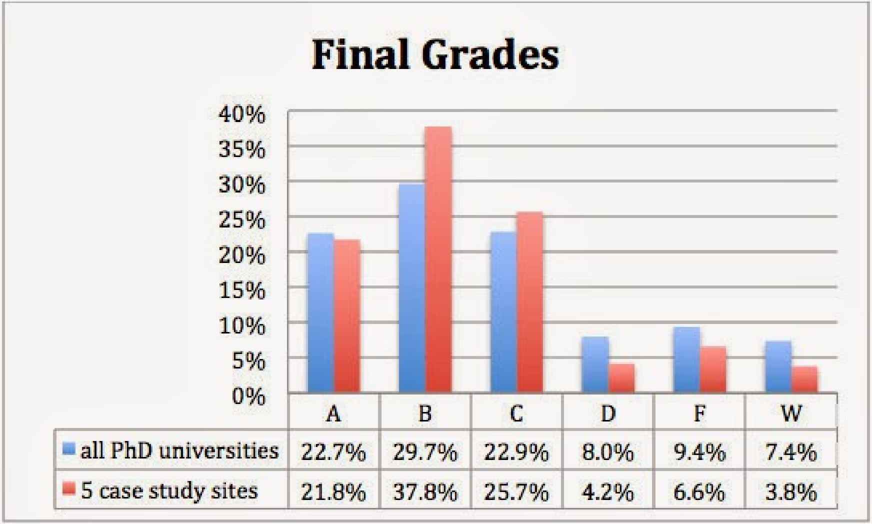
Topological proof 1955

Define sets of the form $A_{j,n} = \{i : i = j \mod n\}$ to be open
Sets of the form $B_p = \{i : i = 0 \mod p\}$ are closed, it is $(\cup_{0<j<p} A_{j,p})^c$
$\{-1, 1\} = (\cup B_p)^c$ is open if there are only finitely many primes.
But all non-trivial open sets are infinite.

Accommodations

- The letter isn't the request. It is a basis for discussion.
- Unlimited Excused Absences. One extra excused absence.

http://www.maa.org/CSPCC



	A	B	C	D	F	W
all PhD universities	22.7%	29.7%	22.9%	8.0%	9.4%	7.4%
5 case study sites	21.8%	37.8%	25.7%	4.2%	6.6%	3.8%

Figure 1: Instructor reported final grades.

Euler and the zeta function $\zeta(1)$

$$\left(1 + 1/2 + 1/2^2 + 1/2^3 + \cdots\right)$$
$$\left(1 + 1/3 + 1/3^2 + 1/3^3 + \cdots\right)$$
$$\left(1 + 1/5 + 1/5^2 + 1/5^3 + \cdots\right)$$
$$\left(1 + 1/7 + 1/7^2 + 1/7^3 + \cdots\right)$$
$$\vdots$$
$$= 1/1 + 1/2 + \cdots + 1/84 + \cdots$$

Punch line

If there are finitely many primes

$$\infty > \prod_p \frac{1}{1-\frac{1}{p}} = \prod_p \left(1 + \frac{1}{p} + \frac{1}{p^2} + \cdots\right) = \sum_n \frac{1}{n} = \infty$$

Euler and the zeta function $\zeta(2)$

$$\left(1 + 1/2^2 + 1/2^4 + 1/2^6 + \cdots\right)$$
$$\left(1 + 1/3^2 + 1/3^4 + 1/3^6 + \cdots\right)$$
$$\left(1 + 1/5^2 + 1/5^4 + 1/5^6 + \cdots\right)$$
$$\left(1 + 1/7^2 + 1/7^4 + 1/7^6 + \cdots\right)$$
$$\vdots$$
$$= 1/1 + 1/2^2 + \cdots + 1/84^2 + \cdots$$

Punch line

$$\prod_p \frac{1}{1-\frac{1}{p^2}} = \prod_P \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right) = \sum_n \frac{1}{n^2} = \frac{\pi^2}{6}$$

If there are finitely many primes the LHS is rational, but the RHS is transcendental (hence irrational).

Appendix $\sum 1/p$ diverges

$$
\begin{aligned}
-\ln(1-x) &= x + x^2/2 + x^3/3 + x^4/4 + \cdots \\
\prod(1/(1-1/p)) &= \sum 1/n = \infty \\
-\sum \ln(1-1/p) &= \sum(1/p + 1/2p^2 + 1/3p^3 + \cdots) \\
&= \sum(1/p) + \sum((1/p^2)(1/2 + 1/3p + \cdots)) \\
&< \sum(1/p) + \sum((1/p^2)(1/ + 1/p + \cdots)) \\
&= \sum(1/p) + \sum((1/p^2)(1/(1-1/p)) \\
&= \sum(1/p) + \sum(1/(p(p-1)) \\
&= \sum(1/p) + C
\end{aligned}
$$

Finally

You have a lot of support, if you need help, ask.
You are the math department.