Fall 2016 Welcome

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Generic properties and friends

- A Generic Property holds on an open dense set.
  - invertible $n \times n$ matrices
  - $n \times n$ matrices with n distinct eigenvalues
  - diagonalizable matrices?

Talking Points

Grade Distributions
Email
Accommodations
ALEKS
Auditors

"First Time in College" students in mac1114, mac1140, mac2233, mac2311 and mac2312 are required to take aleks.
And they must use the FSU Summer/Fall 2016 cohort.
NOT a way to jump from MAC1105 to MAC2311
NOT a way to avoid repeating a course
NOT a way to avoid trigonometry – separate trig score

Auditors

Student Central is now putting auditors into the class roster with a grade basis of "auditor".
Auditors can take tests which have to be graded.
More auditors than usual.

Invertible $n \times n$ matrices is open

$$A^{-1} \text{ exists} \iff \det(A) \neq 0$$

The determinate is a polynomial on n^2 variables and so it is continuous. Thus the inverse image of $\{x \neq 0\}$ is open.

Email

Advisors (other than Danielle or Kari) are not your friend

- Do not reply to email from students wanting to add your class, just forward them to advisor@math.fsu.edu

Invertible $n \times n$ matrices is dense

Suppose $\det A = 0$ and consider $p(t) = \det(A + tI)$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\det(A + tI) = \det(A) + t \left(\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \right) + t^2(a_{33} + a_{22} + a_{11}) + t^3$$

Near zero, $p(t) \sim t^k$ some $k, 1 \leq k \leq 3$ and $A + tI$ is invertible.

Has n distinct eigenvalues is dense

Each A is similar to an upper diagonal matrix U, $A = PUP^{-1}$

$$U = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ 0 & u_{22} & \dots & u_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & u_{nn} \end{bmatrix}$$

whose eigenvalues are $\{u_{ii}\}$, we can perturb these to make U' with distinct eigenvalues and $A' = PU'P^{-1}$ will also have distinct eigenvalues.

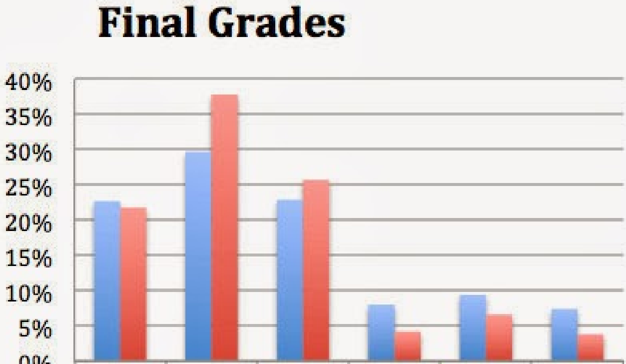
Accommodations

- The letter isn't the request. It is a basis for discussion.
- Unlimited Excused Absences. One extra excused absence.

Has n distinct eigenvalues is open

The coefficents of the characteristic polynomial $p(\lambda) = \det(A - \lambda I)$ is a continuous functon of the entries of A.
Rouché's theorem implies that if p has distinct zero's then there is a $\delta > 0$ so if the coefficents of q are within δ of those in p, then q has distinct roots.
Furthermore if p roots are real, then so are the roots of q.

http://www.maa.org/CSPCC



	A	B	C	D	F	W
all PhD universities	22.7%	29.7%	22.9%	8.0%	9.4%	7.4%
5 case study sites	21.8%	37.8%	25.7%	4.2%	6.6%	3.8%

Figure 1: Instructor reported final grades.

Wilkinson's Polynomial

$$w(x) = \prod_{i=1}^{20}(x - i) = (x - 1)(x - 2)\cdots(x - 20)$$

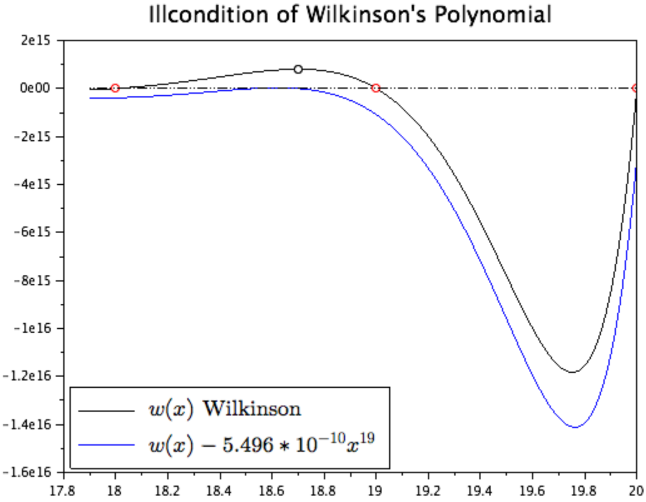
$$\begin{aligned} w(x) = x^{20} &- 210x^{19} + 20615x^{18} - 1256850x^{17} + 53327946x^{16} \\ &- 1672280820x^{15} + 40171771630x^{14} - 756111184500x^{13} \\ &+ 11310276995381x^{12} - 135585182899530x^{11} \\ &+ 1307535010540395x^{10} - 10142299865511450x^{9} \\ &+ 63030812099294896x^{8} - 311333643161390640x^{7} \\ &+ 1206647803780373360x^{6} - 3599979517947607200x^{5} \\ &+ 8037811822645051776x^{4} - 12870931245150988800x^{3} \\ &+ 13803759753640704000x^{2} - 8752948036761600000x \\ &+ 2432902008176640000 \end{aligned}$$

Ill Condition

Decrease the coefficent of x^{19} by a factor of $-210(2^{-31}) \sim -10^{-7}$ to -210.0000001192 and the roots become

1.0000	2.0000	3.0000	4.0000	5.0000
6.0000	6.9997	8.0073	8.9172	20.8469
$10.0953 \pm 0.6435i$	$11.7936 \pm 1.6523i$	$13.9924 \pm 2.5188i$	$16.7307 \pm 2.8126i$	$19.5024 \pm 1.9403i$

A double root appears



diagonalizable is not open

$$A = \begin{bmatrix} 0 & \varepsilon \\ 0 & 0 \end{bmatrix}$$

is not diagonalizable.

Finally

You have a lot of support, if you need help, ask.
You are the math department.