Square or Round

This welcome is brought to you
by Trisecting an angle,
Squareing a Circle,
and Turning a Needle

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The talk is a slide show. The slides are framed in yellow rectangles. The quotation that follows, is what might have been said while the audience was looking at the slide. The blue comments like this one were added later and not part of the welcome. The title frame above was not the original.

Classic Greek Unsolved Problems

Can you using only a compass and an unmarked straight edge:

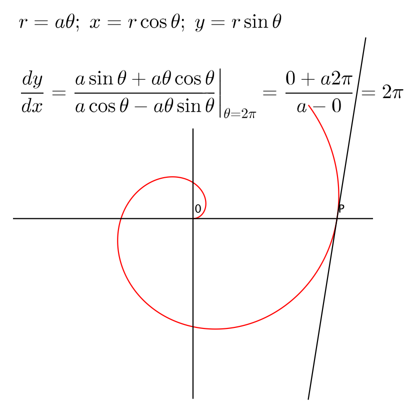
- Trisecting an angle (impossible Wantzel 1837)
- Doubling a cube (impossible Wantzel 1837)
- Square a circle (impossible Lindemann 1880, π transcendental)

There are crazy "circle-squarers" and "angle-trisectors" to this day. Apparently "cube-doublers" were more popular in ancient Greek times.

These are the classical Greek problems that cannot be done with a ruler and straight edge.
Very likely, Archimedes could solve each of these by using addition tools.

Trisecting 60° would construct $\cos 20^\circ$ which has minimal polynomial $8x^3 - 6x + 1$. Doubling the cube would construct $\sqrt[3]{2}$ which has minimal polynomial $x^3 - 2$. Constructable lengths cannot have mimimal polynomials of degree 3. Squaring the circle would construct $\sqrt{\pi}$, but if $\sqrt{\pi}$ is algebraic, then so is $\pi = \sqrt{\pi}\sqrt{\pi}$.

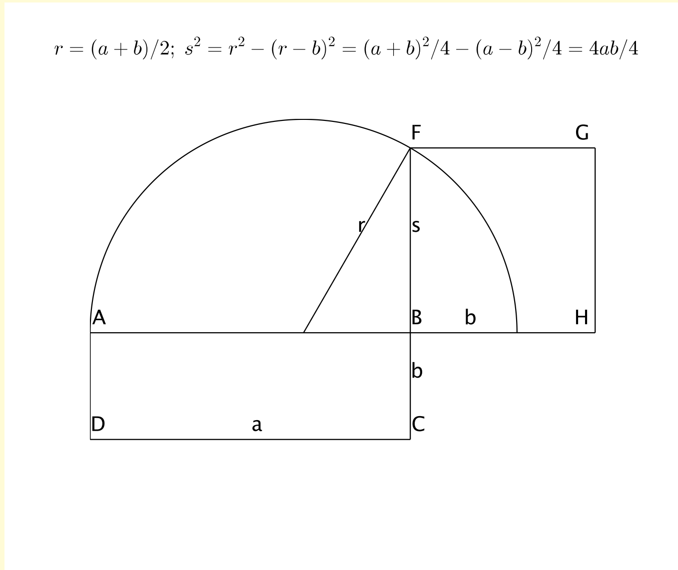
Archimedes Spiral



The curve $r = a\theta$ is an Archimedes spiral. Using the rectangular form, the parametric equations allow us to compute the slope of the tangent at the point $(a, 0)$ (when $\theta = 2\pi$). Let $a = 1$, then the tangent crosses the y-axis at $(0, -2\pi)$ and the area of the triangle form by the two axis and the tangent have area π.

One can draw a spiral by wrapping a string with a pencil at the end about a cylinderical post and drawing while unwinding. Tangents might be hard to line up by eye, but we can imagine it is doable.

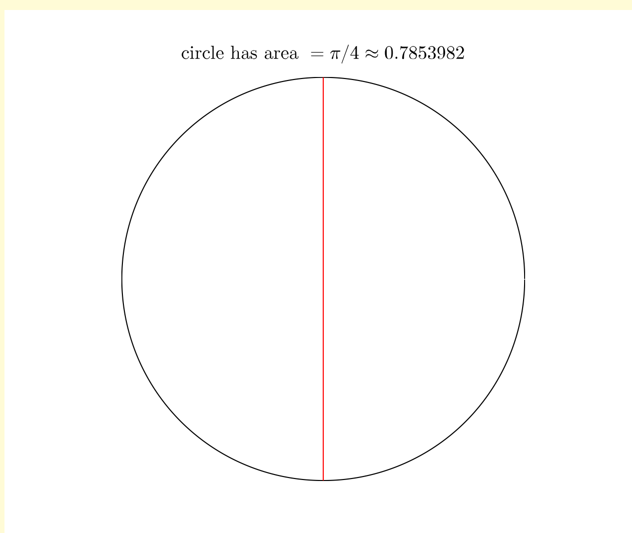
Triangle-squaring



To finish squaring a circle, we need to square the right triangle in the last slide. We convert the area of the triangle, to a rectangular by making the width $1/2$, but keeping the height 2π.

The base of the semi-circle is $a + b$ which allows us to compute r. Pythgorean theorem allows us to compute s^2 which has the same area as the rectangle, namely π.

U Turns

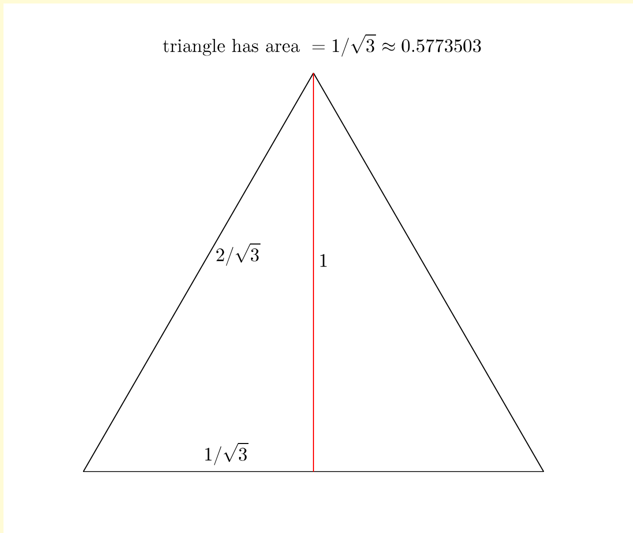


A disk of radius $1/2$ has the property that a needle of unit lenght can rotated inside it so it points in the opposite direction.

Does the circle have the smallest area of all figures with this property? (How about among convex figures?)

We change pace and try to find the smallest area which allows a needle to turn 180°. An obvious example is a circle. One needs to make a jesture which mimics spinning the needle in a circle.

Convex U Turns

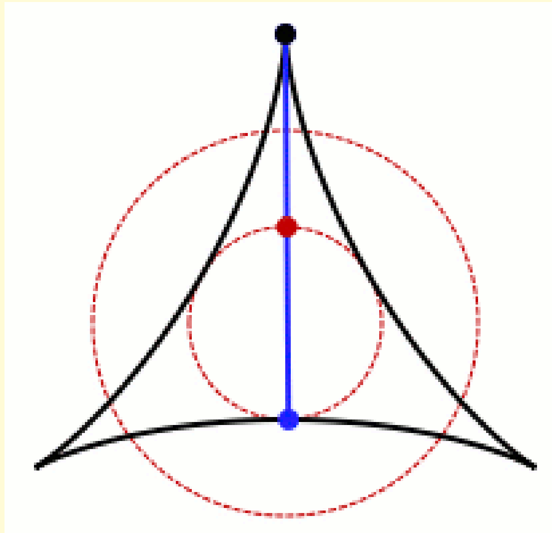


Has the smallest area among convex figures.

The equilateral triangle works too. Rotate to one side, slide to vertex, rotate to bottome edge, slidt to other vertex, rotate to other side, slide to top vertex and finally rotate to the middle. This is an improvement of 25%.

Pai in 1920 proved this was the best one could do with convex regions.

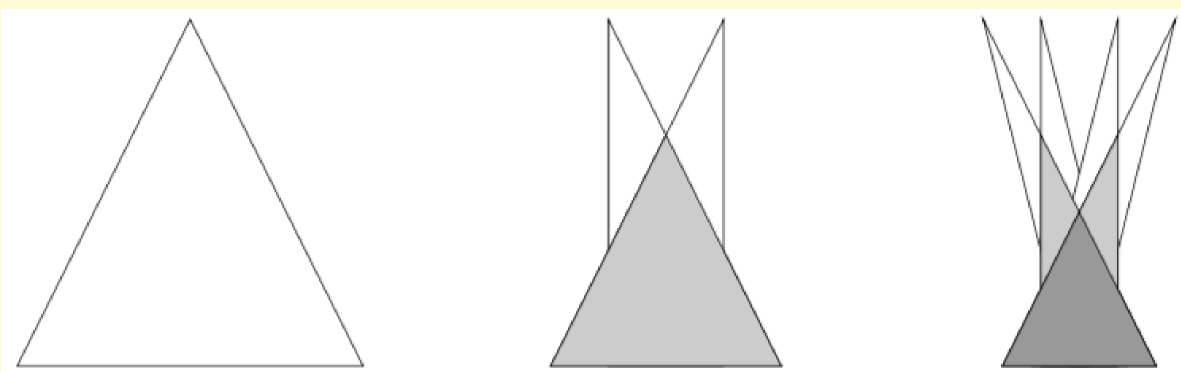
Smaller non-convex I



Area = 0.3853

This picture improves the area another 33% using similar motions with this non-cnvex region called a deltoid.

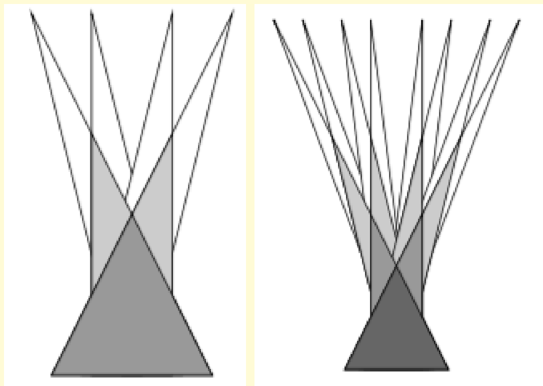
Kakeya (or Besicovitch) Set



That is what I said.

new stuff

Dot dot dot



That is what I said.

I also show the poster below as part of the welcome. Afterwards I realized that all my Teachinng Assistants were not yet born in 1989. I was reminded of this topic at mathfest 2018 in Denver.

Picture sources

before pictures by the author using Scilab.

deltoid picture is from Wikipedia
`https://en.wikipedia.org/wiki/Kakeya_set`

after by the author using screen shots from the following poster created using naked postscript.

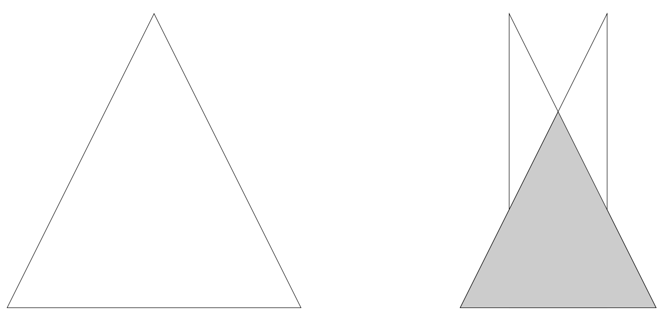
The Analysis Seminar

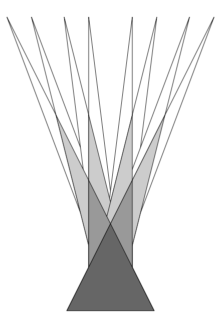
presents

Professor D. McMichael will continue speaking on

“The Kakeya Needle Problem and an

Application to Harmonic Analysis”





3:30 pm Monday

30 October 1989

102 Love