Fall 2019 Welcome

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Talking Points

Eligibility/ALEKS – walk randomly
Email – wait forever
Accommodations – formal power series
Grade Distributions – return with probability one

Random walk on $\mathbb{Z}$ starting at zero

If you are at state i, you flip a coin to decide if you move up or down.

- With probability one, returns to zero infinitely often.
- But the expected return time is never (infinity).
- It is like waiting for Godot, a play that suggests life is full of suffering (written in French)

ALEKS

"First Time in College" students in MAC1105 (and 1114, 1140, 2233 and 2311) are required to take aleks. And they must use the FSU Summer 19 – Spring 20 cohort.

Students with dual enrolled credit, even with AAs are considered FTC.

Not all college courses are equivalent. They need ALEKS for its *inventory of math skills*. And because it provides a *way to improve any weakness* it finds.

NOT a way to jump, avoid repeating, avoid trig

Expect me at the same time

Let x_i be the expected time (number of steps) starting at i to reach 0. From i, it is equally likely to go to $i \pm 1$ so

$$x_i = \frac{1}{2}(1 + x_{i-1}) + \frac{1}{2}(1 + x_{i+1})$$

$$-\frac{1}{2}x_{i-1} + x_i - \frac{1}{2}x_{i+1} = 1$$

Two special cases, by symmetry $x_1 = x_{-1}$ which gives $x_0 - x_1 = 1$ And for $i = 1$ we use 0 instead of x_0 since we have arrived! $x_1 - \frac{1}{2}x_2 = 1$

Solve Me

$$\left[\begin{array}{cccccccc|c}
1 & -1 & 0 & 0 & 0 & 0 & 0 & \dots & 1 \\
0 & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & \dots & 1 \\
0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 & \dots & 1 \\
0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & \dots & 1 \\
0 & 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & \dots & 1 \\
0 & 0 & 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & \dots & 1 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 & \dots & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots
\end{array}\right]$$

Email

Advisors (other than Jennifer or Elizabeth) are not your friend

- Do not reply to email from students wanting to add your class, just forward them to advisor@math.fsu.edu

Upper Triangular Me

$$\left[\begin{array}{cccccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 0 & \dots & 1 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 & 0 & 0 & \dots & 2 \\ 0 & 0 & 0 & 1 & -\frac{3}{4} & 0 & 0 & \dots & 3 \\ 0 & 0 & 0 & 0 & 1 & -\frac{4}{5} & 0 & \dots & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{5}{6} & \dots & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots & 6 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \end{array}\right]$$

To infinity (and beyond?)

$$x_0 = 1 + x_1$$

$$x_0 = 1 + (1 + \frac{1}{2}x_2) = 2 + \frac{1}{2}x_2$$

$$x_0 = 2 + \frac{1}{2}(2 + \frac{2}{3}x_3) = 3 + \frac{1}{3}x_3$$

$$x_0 = 3 + \frac{1}{3}(3 + \frac{3}{4}x_4) = 4 + \frac{1}{4}x_4$$

$$x_0 = 4 + \frac{1}{4}(4 + \frac{4}{5}x_5) = 5 + \frac{1}{5}x_5$$

$$x_0 = 5 + \frac{1}{5}(5 + \frac{5}{6}x_6) = 6 + \frac{1}{6}x_6$$

Accommodations

- The letter isn't the request. It is a basis for discussion.
- Extra time, only at the SDRC
- Notetaker, send email to class, asking them to directly contact the SDRC.
- Anything else for TAs, should be run through either Kirby or Bellenot.

A Formal Power Series

Let p_n be probability the walk is at 0 at the n-th step. Since we start at 0, $p_0 = 1$.
Let f_n be the probability that the first return to 0 is at n (The first passage time). The word return implies $f_0 = 0$.

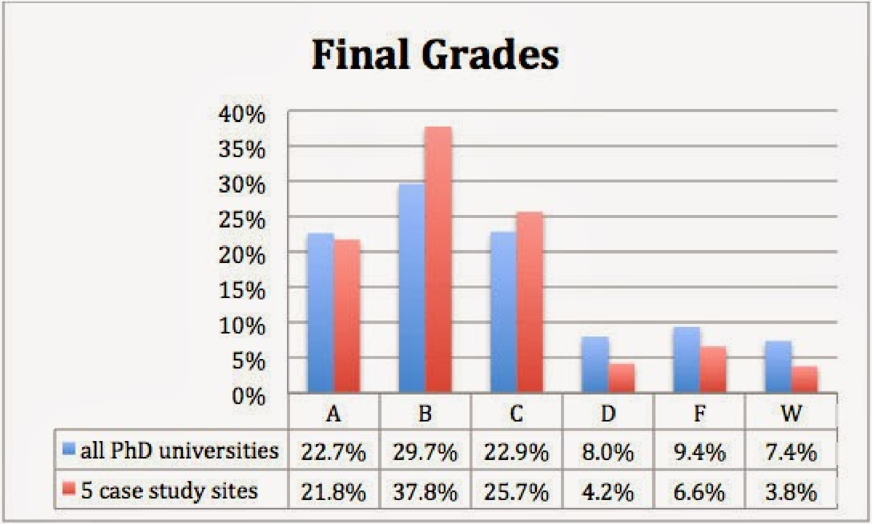
$$P(x) = \sum_{n=0}^{\infty} p_n x^n, F(x) = \sum_{n=1}^{\infty} f_n x^n$$

$$P(x) = 1 + F(x)P(x) \text{ formally } P(x) = \frac{1}{1 - F(x)}$$

$F(1) \leq 1$, If $F(1) < 1$, then $P(1) < \infty$ and if $P(1) = \infty$ then $F(1) = 1$.

Grade Distributions

http://www.maa.org/CSPCC



	A	B	C	D	F	W
all PhD universities	22.7%	29.7%	22.9%	8.0%	9.4%	7.4%
5 case study sites	21.8%	37.8%	25.7%	4.2%	6.6%	3.8%

Figure 1: Instructor reported final grades.

Tangent Line Approximation

$$\frac{\sqrt{n}}{\sqrt{n+1}} = \sqrt{1 - \frac{1}{n+1}} \doteq 1 - \frac{1}{2}\frac{1}{n+1}$$

$$\frac{n+1/2}{n+1} = 1 - \frac{1}{2}\frac{1}{n+1}$$

Summing it up

$$p_{2n} = \binom{2n}{n} \frac{1}{2^n} \frac{1}{2^n}$$

(n steps to the left and n steps to the right in any order).

$$\binom{2n}{n} \sim \frac{4^n}{2\sqrt{n}}$$

Induction, $n = 1$, $\binom{2}{1} = 2 = \frac{4^1}{2\sqrt{1}}$

$$\binom{2(n+1)}{n+1} = \frac{(2n+2)!}{(n+1)!(n+1)!}$$

$$= \frac{(2n+2)(2n+1)}{(n+1)(n+1)} \binom{2n}{n} = \frac{2(2n+1)}{n+1} \binom{2n}{n}$$

$$= 4 \frac{n+1/2}{n+1} \binom{2n}{n} \doteq 4 \frac{\sqrt{n}}{\sqrt{n+1}} \binom{2n}{n} = \frac{4^{n+1}}{2\sqrt{n+1}}$$

Summing it up

This $p_{2n} \sim 1/\sqrt{n}$ and $\sum p_n = \infty$ and the walk returns with probability one.

Finally

You have a lot of support, if you need help, ask.
You are the math department.