Using an Ecological Spatial Stochastic Model as an introduction to Modeling and Scilab

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Target Audience

- Course is a 1-hour MAP 2480 Biocalculs Lab
- Biology majors, many Pre-Med
- Calculus I: MAC 2311 pre-requisite
- Some students are 3-4 years from Calculus
- Pre-meds work for grades, cram instead of learning
- No programming experience

Teaser Question for this Ecology Lab

- Why are there so many different species?
- We give evidence as to why this is a hard question, we don't answer it.
- In fact, for each kind of resource, there is only one species using that resource.
- Diversity of species implies diversity of resources.

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Life on a checkerboard

- Plants grow on a checkerboard
- Each square has one plant
- Reproduction via nearest neighbors
- Repeat:
  - select plant
  - Voter Model, plant is replaced by a neighbor's offspring
  - Invasion Process, plant's offspring replaces a neighbor
- generation is nm births for $n \times m$ checkerboard.

The Colormap

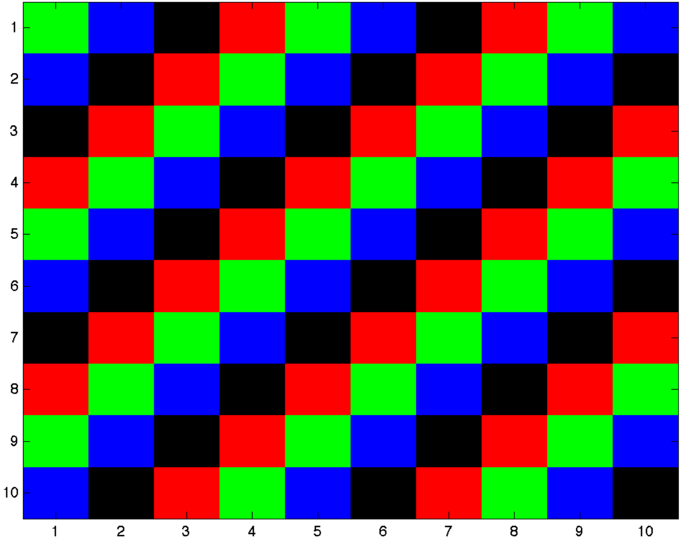
Species	Image Color
1	black
2	red
3	green
4	blue
5	yellow
6	magenta
7	cyan

The colormap installed by init.m; (careful 0 is also black).

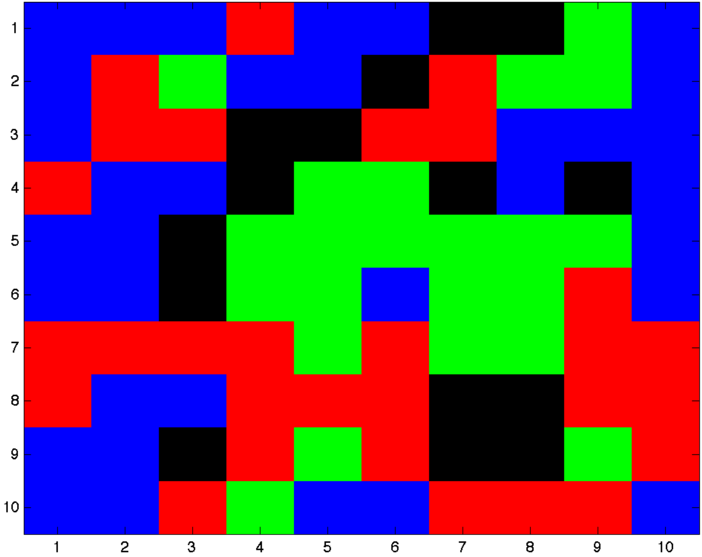
Initialization Code

```
n=10; A = zeros(n,n);
for i=1:n, for j=1:n,
A(i,j) = mod(i+j,4)+1;
end; end;
cm = [ 0 0 0; % 1 black
1 0 0; % 2 red
0 1 0; % 3 green
0 0 1; % 4 blue
1 1 0; % 5 yellow
1 0 1; % 6 magenta
0 1 1 ]; % 7 cyan
colormap(cm);
% 0 also gets black (why the plus one)
```

Initial Position



One generation later



Sample Code

```
init;
image(A); % Scilab uses Matplot
rand('seed', 1234);
A = generationip(A);
image(A);
```

To get a movie:

```
init; for i=1:100;
A=generationip(A); image(A); pause(0.01);
end;
```

Observations

Experience with this model shows two behaviors:

1. The colors blotch together; plants of the same color tend to clump or cluster together.
2. Eventually all rectangles have the same color, one species wins and the others die out.

The second model

Suppose there were at most 2 species and we consider only the population of species 1. Each event in our old model did one of there things:

1. A plant was replaced by the same species. Popuation change: 0
2. A plant of species 1 was replaced by species 2. Population change: -1
3. A plant of species 2 was replaced by species 1. Population change: +1

And the last two events are equally likely. (Chose the edge and the then the direction.)

Dunkard's walk on 2×2

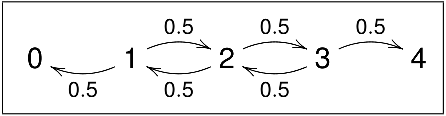


Figure: Diagram showing the transitions with probabilities

$$T = \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 1 \end{bmatrix}$$

Markov Chain

If $X = \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_n \end{bmatrix}$ with $x_i \geq 0$ and $\sum x_i = 1$, then as $n \to \infty$, $T^n X \to \begin{bmatrix} s & 0 & 0 & \dots & 1-s \end{bmatrix}$ some s, $0 \leq s \leq 1$. Fixed points of T, eigenvectors for the eigenvalue 1.

Grading Labs

We have 200 students this semester, in six sections of 33 students.
98% of the class is computer graded via MapleTA. The lab is a mastery assignment, they must get question n right before moving on to question $n+1$.
The TA's actually want to grade 2%. It gives they a better sense of who is in the class.

Lab Question 0

This question exists to make sure you pay attention to the lecture at the beginning of lab. Your number is x; eventually your instructor will tell you how to answer this question.

Answer: $x \mod 13$

Lab Questions (cont)

1. For loop output (for $i = a : b$, i^n, end;)
2. Next random number after seeding
3. Who is the winner after seeding
4. The movie – oracle question
5. Two steps for the dunk
6. 3 or 4 for the dunk
7. limit $T^n X$
8. Movie 2: 25x25 and red vs green – oracle question

Oracle Questions

Often we will have complex commands for you to do which are not easily graded on the computer. After you have done the task, as one of the instructors to check your work. If it is correct he will give you the answer to question “What is the answer to question number a?”
Answer: There is a sheet with the answers. The answer is the next random number for a simple-to-compute random number generator.

Additional Directions

- Monte Carlo simulations to estimate time to domination.
- Geographic shapes (islands and land bridges) instead of checkerboards.

Summary

- Stochastic and spacial models provides a more general first view of modeling as compared to curve fiting.
- A nice animation to amuse the students.
- A dunkards walk, monte carlo, islands, and more.

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