Using Scilab to teach ODE Topics

Steven F. Bellenot

Department of Mathematics
Florida State University

Math Tech, Valdosta State University, Valdosta, GA,
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ODE solving in Scilab

Solving the IVP

$$\frac{dy}{dt} = f(t, y) \qquad y(t_0) = y_0$$

- define the function f(t,y)
- time steps: ts = t0:delta:tf;
- ode solver: y = ode(y0, t0, ts, f);
- solves $\frac{dy}{dt} = f(t, y), \quad y0 = y(t0)$
- the output y has values for t at the time steps given by ts

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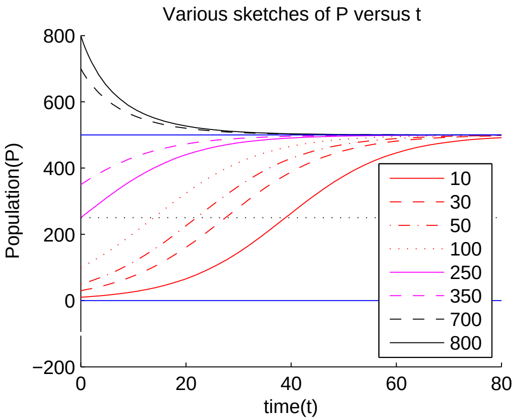
Logistic Population Growth

$$\frac{dP}{dt} = rP(1 - \frac{P}{K})$$

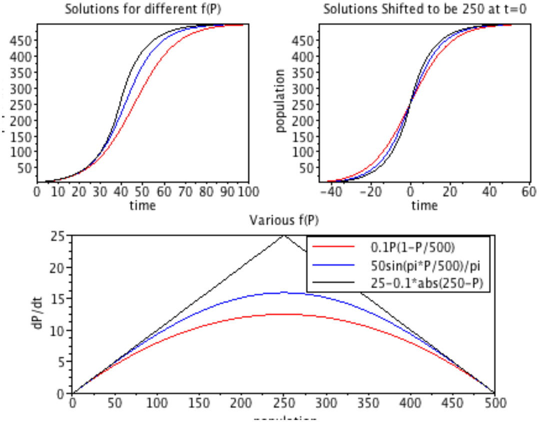
```
function dPdt = logistic(t, P)
dPdt = rate * P .* (1 - P/capacity); //dot star
endfunction
rate = 0.1; capacity = 500;
w0 = [10;30;50;100;250;350;700;800];
odeCheckPlot(w0,0,0:80,logistic);
```

Logistic I



Various sketches of P versus t
800
600
400
200
0
−200
Population(P)
0
20
40
60
80
time(t)
10
30
50
100
250
350
700
800

Logistic II Student Problem

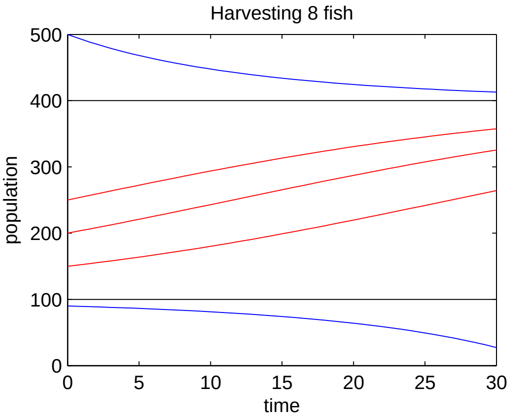


Logistic with Harvesting

$$\frac{dP}{dt} = rP(1 - \frac{P}{K}) - h$$

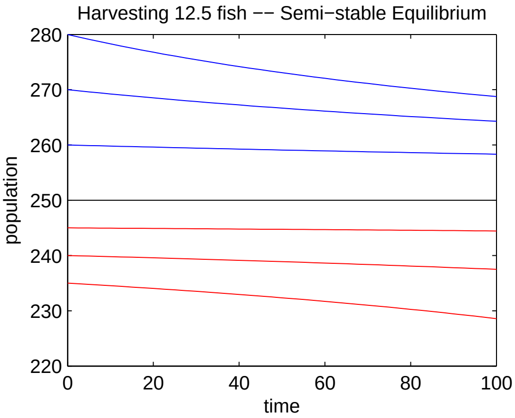
By adding harvesting the topic of stability of equilibrium solutions arises naturally and examples of stable, unstable and semi-stable solutions all appear.

Harvest I



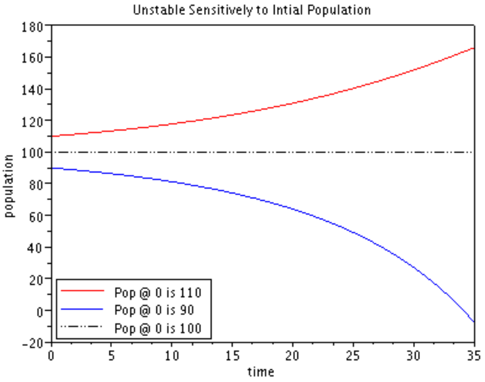
When $h = 8$, $P = 400$ is stable, $P = 100$ is unstable.

Harvest II



When $h = 12.5$, $P = 250$ is semi-stable.

Harvest III Student Problem



SIR Model of an Epidemic

$$\frac{dS}{dt} = -\lambda SI$$

$$\frac{dI}{dt} = \lambda SI - \mu I$$

$$\frac{dR}{dt} = \mu I$$

Inflection peak when $S = \mu/\lambda$ is one measure of the inflection. Another is sick days.

SIR Model of an Epidemic

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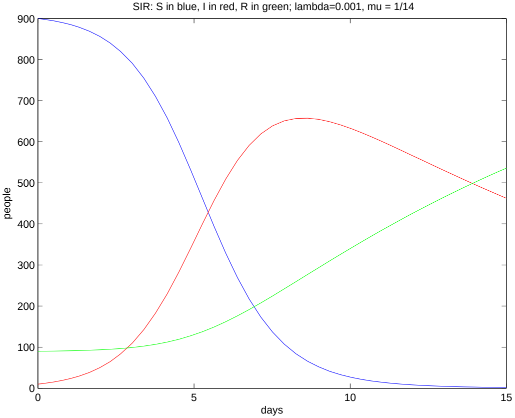
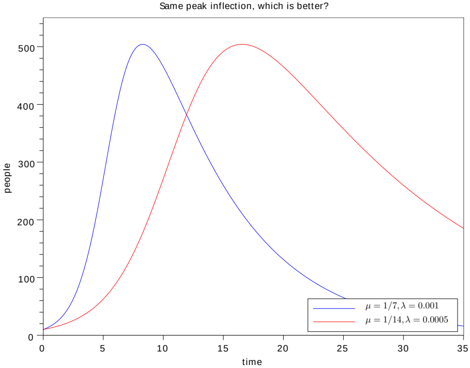
SIR I



SIR: S in blue, I in red, R in green; lambda=0.001, mu = 1/14
people
days
0
100
200
300
400
500
600
700
800
900
0
5
10
15

SIR II Student Problem



Predator Prey, The Lotka-Volterra Model

$$\frac{dC}{dt} = \alpha C - \lambda CR$$
$$\frac{dR}{dt} = -\beta R + \mu CR$$

$C(t)$ Extrema when $R = \alpha/\lambda$, $R(t)$ Extrema when $C = \beta/\mu$

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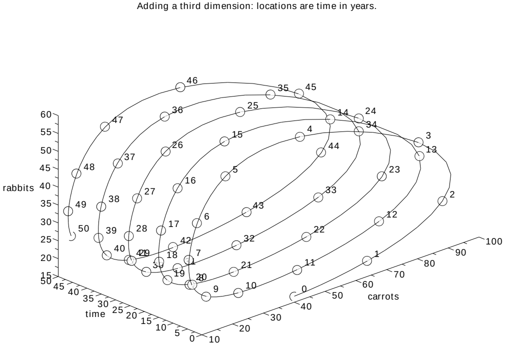
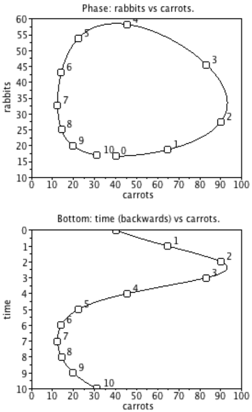
Prey I Phase Space



Prey II Student Problem



Without weed killer

$$\frac{dW}{dt} = r_W W(1 - \frac{W}{100})$$
$$\frac{dH}{dt} = r_H H^2(1 - \frac{H}{100})$$

With weed killer

$$\frac{dW}{dt} = -\beta_W W$$
$$\frac{dH}{dt} = -\beta_H H$$

Gene of time periods where weed killer is applied.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Chemotherapy of the Lawn

Without weed killer

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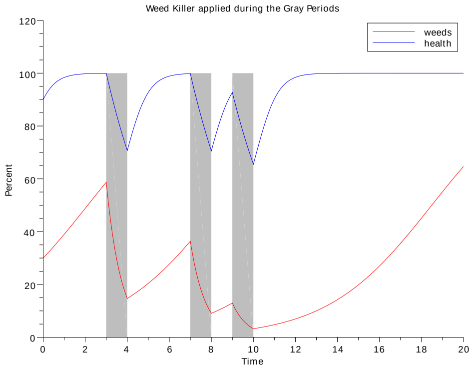
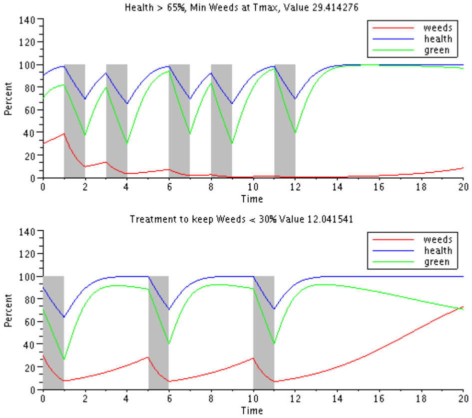
Gene of time periods where weed killer is applied.

$$\left[\begin{array}{cccccccccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right]$$


Weed Killer applied during the Gray Periods
weeds
health
Percent
Time
0
20
40
60
80
100
120
0
2
4
6
8
10
12
14
16
18
20

Chem II Student Problem



2nd Order Example: Damped Pendulum

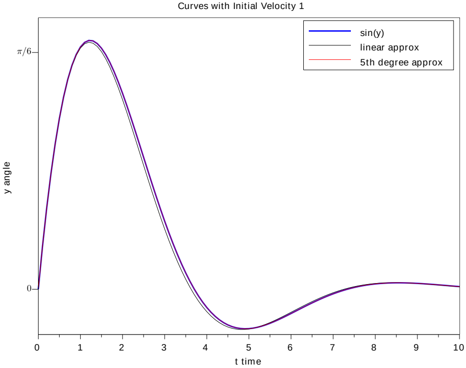
$$y'' + y' + \sin(y) = 0$$

$$y'' + y' + y = 0$$

$$y'' + y' + y - \frac{y^3}{6} = 0$$

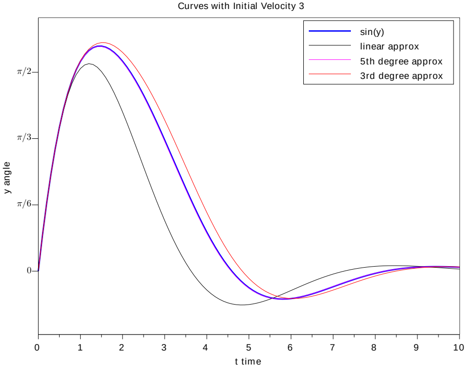
$$y'' + y' + y - \frac{y^3}{6} + \frac{y^5}{120} = 0$$

Swing I



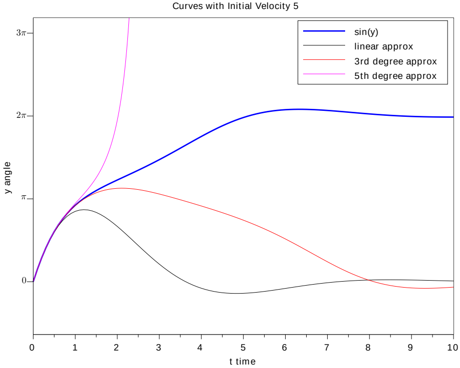
$$y(0) = 0, y'(0) = 1$$

Swing II



$$y(0) = 0, y'(0) = 3$$

Swing III



$$y(0) = 0, y'(0) = 5$$