Introduction to Computational Neuroscience (Spring 2018)

Using Bifurcation Analysis to Examine Tonic Spiking in HVC_{RA} Neurons

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We now return to the HVC_{RA} neuron in current clamp mode, and determine the range of applied current that induces tonic spiking in the cell. We will then determine how this range is effected by some of the channel conductances. The approach we will use is bifurcation analysis.

Exploration

Download the file HVCRA_BD.ode from my web site. This has applied current, I_{ap}, as a parameter rather than as a current pulse.

(1) Start up the code and run it with the default parameters ($I_{ap} = 0$). Try running with a few other values of I_{ap} to get a ballpark idea about the range of I_{ap} that will produce continuous spiking. Then open up AUTO and generate a stationary bifurcation diagram. This diagram should extend far enough so that two Hopf bifurcations appear, which you should record. (You can use the sampling you just did to get an idea about how to set the x min/max and y min/max parameters, as well as the parmin/parmax parameters.) Once this is done, generate a branch of periodic solutions. These represent tonic spiking. Based on the properties of the periodic branch, are the Hopf bifurcations subcritical or supercritical?

(2) Redo the stationary branch, but using a smaller leak conductance of $g_L = 4$ nS rather than $g_L = 7$ nS. Now where are the Hopf bifurcations? Does tonic spiking begin at lower or higher values of the applied current? Is the tonic spiking range larger or smaller now? Why does this happen?

(3) Tonic spiking occurs for values of I_{ap} between the two Hopf bifurcations, and as we have seen this is different for different values of the leak conductance. Rather than constructing one-parameter bifurcation diagrams for different g_L values and looking for the Hopf bifurcations, one could do a one-parameter diagram once, then grab the Hopf bifurcation and do a **two-parameter bifurcation diagram**, where a curve of Hopf bifurcations is generated in the two-parameter plane of I_{ap} (x-axis) and g_L (y-axis). This curve then summarizes how the I_{ap} value of the Hopf bifurcation varies with changes in g_L. You could do this with either or both Hopf bifurcations. To do a two-parameter bifurcation diagram once you have already made your one-parameter diagram

(the one you just made), click on *Axes* and then *two par*. This will open up a new window. The first entry is not used, so what matters are the others. You should see that I_{ap} is listed as the main parameter and g_L as the second parameter. For the range of I_{ap} you can use what you have been using. For the range of g_L you will need to make a guess, and probably modify it later. After setting the ranges, click OK. Next, grab one of the Hopf bifurcations and click on *Run* and *Two Param*. When you do this you should see a curve traced out in the I_{ap}-g_L plane. This is a curve of Hopf bifurcations. (If you don't see anything it could be because your param max is not big enough in Numerics, or because your viewing range is not appropriate.) Does the change in the I_{ap} value of the Hopf bifurcation with a change in g_L agree with your work from the previous problems?

(4) In the previous problem you continued the Hopf bifurcation in one direction, probably using a positive Ds value in AUTO. To complete the curve try grabbing the same point as before, but setting Ds negative in Numerics before clicking on Run. The complete curve should look like a slanted parabola. If it does not, then you have not continued far enough. You may need to increase your Par Max in the Numerics window.

(5) One way to view the two-parameter bifurcation diagram is in terms of slices. A horizontal slice through the diagram, in which g_L is fixed at some value, would have two points if $g_L = 4$ nS, which are the left and right Hopf bifurcations seen in the earlier one-parameter bifurcation diagram. The distance between these points is the range of tonic spiking. How does that range vary as g_L is increased? The curve only exists for g_L sufficiently small. What is the critical value of g_L? What happens to the Hopf bifurcations at that critical value? What does the one-parameter diagram (with parameter I_{ap}) look like beyond the critical value of g_L? What region of the two-parameter plane supports tonic spiking?