

Assignment 1 (Graph Theory and Networks)

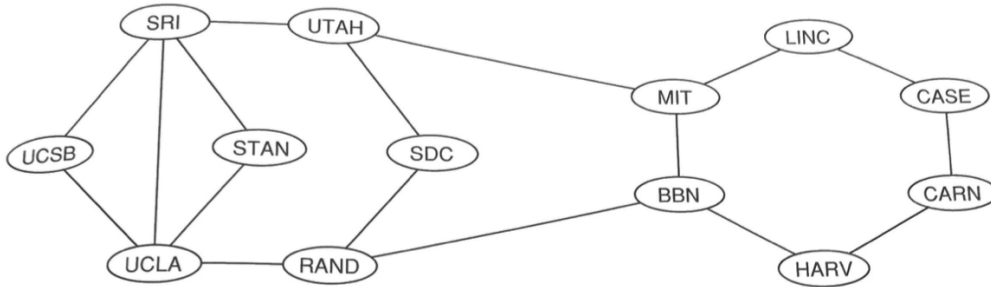
Due on September 12

- (1) Suppose that you work for Netflix and you notice the following:
- Richard rented Day of the Dragon Fly and The Revenge of the Crickets and Spider Man 17: The Last Chapter
 - Sophie rented All Roads Lead to Perry and Spider Man 18: Just Kidding
 - Noel rented Day of the Dragon Fly and All Roads Lead to Perry
 - Miles rented The Revenge of the Crickets and Spider Man 17: The Last Chapter

Represent this as a bipartite graph and its two one-mode projections. Based on these networks, make a recommendation for Sophie and explain your reasoning.

- (2) A node X is called *pivotal* for a pair of distinct nodes Y and Z if X lies on every shortest path between Y and Z (and X is not equal to Y or Z).
- (a) Give an example of a graph in which every node is pivotal for at least one pair of nodes. Explain your answer.
- (b) Give an example of a graph having at least four nodes in which there exists a single node X that is pivotal for every pair of nodes (not counting pairs that include X). Explain your answer.
- (3) Suppose that A and B are vertices in a network and that you can identify two node-independent paths between them. Is it always the case that if you pick one vertex in each path at random and remove it (and edges connected to it) the vertices will be disconnected? If so, explain why. If not, give an example.

- (4) List all the cycles involving MIT in the network below of the ARPANET (each node is a computer server at a university). Which edge, or edges, would have the largest impact on the number of cycles involving MIT if it were removed?



- (5) The node degrees can be collected into a vector \vec{k} .
- Suppose that the degree vector for a simple undirected (and unweighted) network is $\vec{k} = (2, 2, 3, 3, 2)$. Give a plausible adjacency matrix \mathbf{A} for the network.
 - Next consider a simple directed network with in-degree and out-degree vectors $\vec{k}^{\text{in}} = [3, 3, 2, 3, 2]$ and $\vec{k}^{\text{out}} = (3, 3, 3, 3, 1)$. Give a plausible adjacency matrix \mathbf{B} .
 - What is the network density for each of these networks?