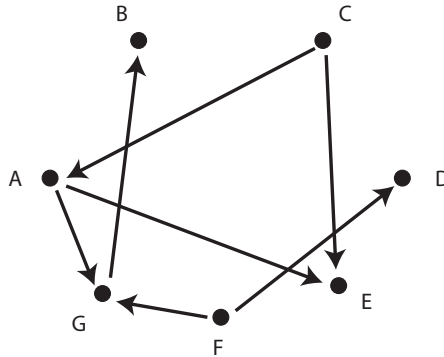


## Assignment 2 (Graph Theory and Networks)

Due on September 26

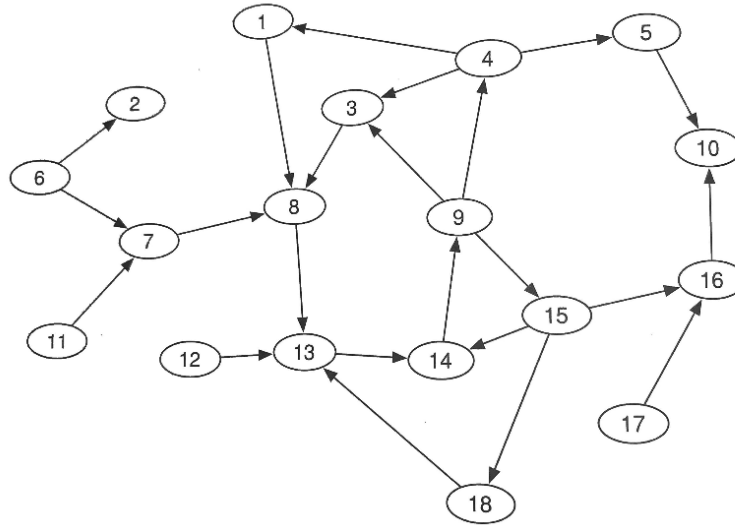
- (1) Prove that the network below is acyclic by re-arranging so that all edges point downward, and then relabeling vertices so that the adjacency matrix is strictly lower triangular.



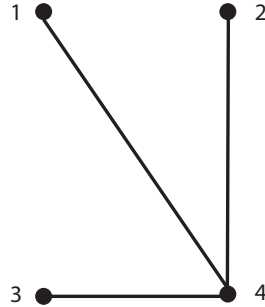
- (2) Can you determine the bibliographic coupling network solely from the cocitation network? In terms of matrices, can you determine the adjacency matrix for bibliographic coupling,  $\mathbf{B}$ , from the cocitation adjacency matrix,  $\mathbf{C}$ ? If yes, explain how. If no, give an example.
- (3) The World Wide Web has a bowtie structure, which is illustrated in this problem. Consider the set of eighteen Web pages shown in the figure below, with links forming a directed graph.
- Which set of nodes constitutes the largest strongly connected component (SCC) in this network?
  - Taking this as the *giant SCC*, which nodes represent Web pages that have some path into the giant SCC, but are not themselves in the giant SCC? (These constitute the IN set.)
  - Which nodes represent Web pages that the giant SCC links to from some path, but are not part of the giant SCC? (These constitute the OUT set.)
  - Tendrils* consist of (i) nodes that are reachable from IN that cannot reach the giant SCC and (ii) nodes that can reach OUT but cannot be reached from the giant SCC. In the network below, which nodes constitute the

tendrils?

- (e) Name an edge you could add or delete so as to increase the size of the giant SCC.
- (f) Name an edge you could add or delete so as to increase the size of IN.
- (g) Name an edge you could add or delete so as to increase the size of OUT.



- (4) Consider diffusion on the graph shown below, with diffusion coefficient  $c$ . Suppose that the diffusing substance has initial distribution  $\vec{u}(0) = (4, 3, 2, 7)$ .
- (a) Write down the graph Laplacian.
- (b) What is the value of the smallest eigenvalue,  $\lambda_1$ , and its corresponding eivenvector  $\vec{v}_1$ ?
- (c) What is the initial value of the first coefficient of the spectral solution,  $a_1(0)$ ?
- (d) What is the equilibrium distribution of substance across the graph?



- (5) This problem illustrates how filtering can smooth an image. Consider the network below with **affinities** listed along the edges. Suppose that the initial image intensity vector is  $\vec{u}_0 = (2, 74, 150, 1, 18, 46, 3, 200)$ . Write a Scilab code to do the following (print out and turn in a copy of the code). You **don't have to** write down the structures below by hand (e.g., the **D** matrix), just make sure that they are printed out when the script file is run. Then answer the questions based on your Scilab output.
- Compute the degree matrix **D** (you don't need to show this in your answer).
  - Compute the filter matrix **W** (you don't need to show this in your answer).
  - Compute and report the vector  $\vec{u}_{20}$  by iterating with the filter matrix. By doing the iterative filtering, have the intensity values moved apart or have they moved together?
  - Replace all the affinities of 1 with affinities of 0 (this would eliminate the edge). What is  $\vec{u}_{20}$  when iterating with the new filter matrix? Why? (remember to type "clear" in Scilab before re-running your script file.)
  - In this latter case, you should notice that, after 20 iterations, node 1 is different from all the others. If you iterate one more time, then something

happens with the intensity of node 1 and that of nodes 2, 3, and 4. What is happening and why is it happening?

