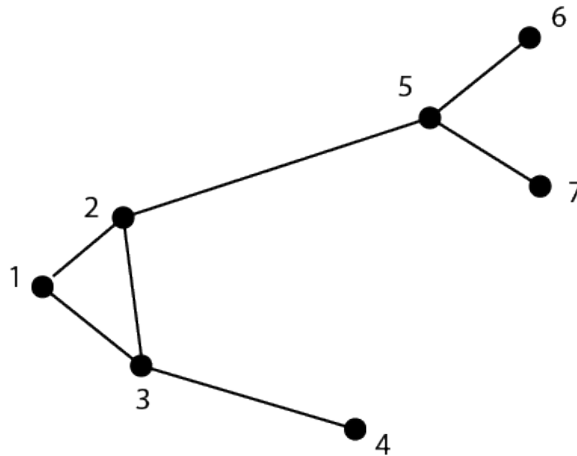


## Assignment 3 (Graph Theory and Networks)

Due on October 14

(1) Consider the undirected graph below. For each question, in case of a tie, answer with all the tied top nodes.

- Which node has the highest degree centrality?
- Which node has the highest betweenness centrality?
- Which node has the highest closeness centrality?

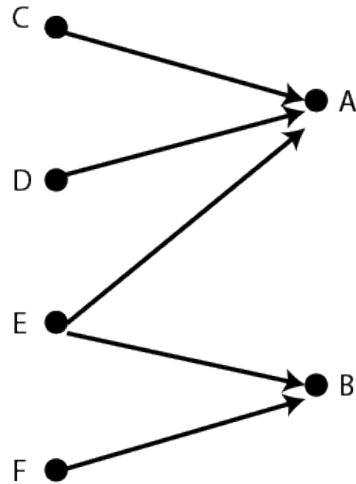


(2) Provide an example of a network such that the node with the highest degree is not the one with the largest closeness.

(3) Web designers strive to find ways in which their Web pages score highly on search engine rankings. This question explores some of the reasoning that is involved. The goal is to create Web pages to achieve large authority scores, given an existing hyperlink structure, which in our example is the directed graph below, where each node represents a Web site.

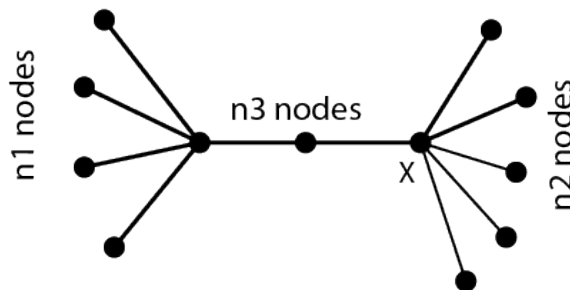
(a) Suppose you want to create a new Web page X and add it to the network so that it could achieve a normalized authority score that is as large as possible. One thing you might try is to create a second page Y as well, so that Y links to X and thus confers some authority to it. You may wonder, though, whether it helps X's authority to have Y link to other nodes as well. Specifically, suppose you add X and Y to the network. Two options are

- Option 1:* Add new nodes X and Y to the network, create a single link from Y to X, and create no links out of X.
- Option 2:* Add new nodes X and Y to the network; create links from Y to each of A, B, and X; and create no links out of X.



For each of these two options, we'd like to know how X fares in terms of its authority score. So, for each option, show the normalized authority values that A, B, and X get when you run two iterations of the hub-authority calculation. For which of options 1 or 2 does page X get a higher normalized authority score? What is a plausible explanation for this result?

- (b) Suppose you create three new Web sites, X, Y, and Z to add to the network and again try to strategically create links out of them so that X gets ranked as well as possible. Describe a strategy for adding three nodes X, Y, and Z to the network with links of your choice so that when you run two iterations of the hub-authority calculation and rank all pages by their normalized authority score, node X shows up in second place. [In this network, there is no way for X to do better than second place when only three nodes are added to the network as described.]
- (4) Consider the network below. In addition to node X, there are  $n_1 + n_2 + n_3$  nodes as indicated. Determine the betweenness and closeness centralities of X.



- (5) In this problem you should write Scilab or Matlab script files (.sce or .m files) to do operations on a network and its adjacency matrix. Turn in a printout of the .sce file along with the written answers to the questions below. The questions are based on the following adjacency matrices for two graphs, one undirected and the other directed.

- (a) Define the following simple, undirected adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

and the following directed adjacency matrix

$$B = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Sketch by hand the two networks that these correspond to.

- (b) Compute the degree of each node in the undirected network. Then compute the in-degree and out-degree for each node in the directed network.
- (c) Compute the number of edges in each network.
- (d) Compute the mean degree ( $c$ ) and density ( $\rho$ ) for the undirected network. Then compute the mean in-degree and mean out-degree of the directed network to verify that they are equal. Finally, compute the density of the directed network.
- (e) Compute the eigenvalues and eigenvectors of  $\mathbf{A}$ . Is this consistent with the Perron-Frobenius theorem?
- (f) Determine the degree centrality and eigenvector centrality for the undirected network. Are the most central vertices the same in each case?