Assignment 4 (Graph Theory and Networks)

Due on November 2

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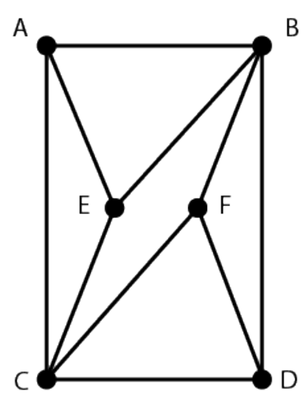
(1) (a) Determine the modularity of a bisection of the complete graph K_5 in which one cluster contains two nodes and the other contains three nodes.

(b) Generalize this for all possible bisections of K_5. That is, give a formula for modularity for a bisection in which one cluster contains N_1 nodes and the other contains the remaining nodes.

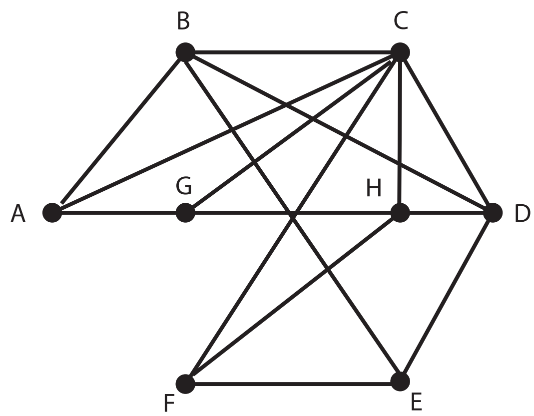
(c) Generalize this further to the complete graph K_N. What is the modularity of this graph when one cluster of the bisection contains N_1 nodes and the other cluster contains the reamaining nodes? Show that this is always negative, regardless of the bisection used. [Since the partition of a single cluster has modularity zero, it is also the maximum modularity partition. In other words, modularity optimization does not split cliques.]

(2) Suppose that A and B are two of the $q > 2$ clusters of a network partition, with degrees k_A and k_B, respectively. Also suppose that these are approximately the same, so $k_A \approx k_B = k$. Let L_A, L_B, and L_{AB} be the number of links inside cluster A, inside cluster B, and between A and B, respectively. Compute the difference in modularity between this partition and the one in which A and B are merged, called cluster C. What condition on k makes the partition with A and B merged have larger modularity than the one in which they are separated?

(3) A bipartite affiliation graph that shows the membership of people in different social foci can be projected into a graph that just shows the people, in which two people who share a common focus are joined by an edge. An example of such a projection is shown below. For this projection, draw a bipartite affiliation network consistent with this projection and having the minimum number of foci possible. Explain how you know that this is the minimum number.



(4) In the social network below, calculate the neighborhood overlap of all edges and place tie strengths on the edges in such a way that greater tie strengths are associated with greater neighborhood overlap. Are there any local bridges? If so, list them.



(5) The complementary cumulative distribution function $\mathbf{P}_k$ of a discrete probability distribution p_k is defined as $\mathbf{P}_k = \sum_k^{\infty} p_k$. Show that the complementary cumulative distribution function of a power-law distribution $p_k = Ck^{-\alpha}$ with exponent $\alpha > 1$ and $k = 1, 2, ...$ is itself a power-law distribution. What is the exponent of the $\mathbf{P}_k$ distribution? (Hint: approximate summation with integration.)