Assignment 5 (Graph Theory and Networks)
Due on November 16

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(1) Consider the following process. First, start with a collection of N nodes and no edges. Then, one by one, add an edge between two nodes not already connected to each other. Continue until you have a complete network.
   (a) How many steps are there in the process?
   (b) Which of the following tends to be true about the sequence of largest component sizes as you add edges? Explain your answer.
      (i) Increases slowly at the beginning of the sequence, increases very fast at the end.
      (ii) Increases slowly until some threshold, increases very quickly for a short time, and then increases slowly afterwards.
      (iii) Increases at a constant rate.
      (iv) Increases very fast at the beginning of the sequence, then tapers off and increases slowly until the end.
      (v) Increases and decreases randomly.

(2) Consider a ring-like lattice like the one used in the Watts-Strogatz model, with no random rewiring. Suppose that there are $N = 100$ nodes and that each node connects with $K = 4$ nearest neighbors; two on one side and two on the other.
   (a) Determine the mean geodesic path length between nodes.
   (b) Add an edge from node n_1 to node n_{50} (the nodes are numbered clockwise with n_1 at the top). What is the new mean geodesic path length from node n_1 to other codes? How does this compare to the mean path length before the extra edge was added?

(3) Create the Scilab file **random_network.sce** (or a similar MATLAB file) that generates the adjacency matrix for a 100-node undirected Erdós-Rényi random network with no self-edges. Use a connection probability of $\phi = 0.2$. You will need to use the "rand()" function to build this matrix, $\mathbf{A}$.
   (a) Demonstrate that the adjacency matrix is symmetric.
   (b) Make a plot of the degree distribution, showing the number of nodes with various degrees. Are there any hubs in the network? Does the network appear to be binomial? What is the largest degree of the network
   (c) How many edges are there, and how does this compare with the expected value of the number of edges from probability theory? ***(For this, and following questions, write down in your report the formulas you used to determine the quantities in question.)***

(d) What is the mean degree, and how does this compare with the mean value expected from probability theory?

(e) Using the actual mean degree, determine the clustering coefficient for the network. How does this compare with the clustering coefficient from probability theory?

(f) Use the graph Laplacian to determine if the network is connected. If not, how many nodes are in the giant component?