Introduction to Computational Neuroscience (Spring 2018)

Bifurcation Analysis of the Morris-Lecar Model

In this exercise we'll construct bifurcation diagrams using the program AUTO, which is implemented in the XPPAUT software package. We'll work with the Morris-Lecar model and vary the applied current. We used the code last time, but if you've misplaced it you can download it from my web site (under Course Software). You don't need to turn in anything for this assignment, just go through it so that you'll learn how to make bifurcation diagrams with AUTO.

Exploration

To construct a bifurcation diagram you must start with an equilibrium point, which can be found using XPP. The AUTO module is then used to continue this single equilibrium to form a curve of equilibria, as the bifurcation parameter is varied.

(1) To invoke AUTO from within XPP click on File and then Auto. This opens up an AUTO window. To construct a bifurcation diagram you first decide on a bifurcation parameter (in our case, I_{ap}) and an initial value of that parameter (in our case 0). Run XPP with that value several times, using Initial Conditions Go (IG) or Initial Conditions Mouse (IM) followed by Initial Last (IL) 2 or 3 times, and watching to see that your variables are very close to a stable equilibrium. IL will read the last values of the variables in as initial conditions, and these are what AUTO will use as the coordinates of the steady state. Right now $I_{ap} = 0$ in your code, so if you run XPP as above then you should get coordinates for the resting steady state.

(2) Next click on Parameter in the AUTO window. Put the name of the parameter that you want to use as your bifurcation parameter as Par1. When done, click OK.

(3) Next click on Axes and HiLo. Par1 is automatically put onto the X-axis. Put one of the variable names as the Y-axis (in this case, put V). Then enter the parameter interval that you want to explore (Xmin and Xmax), and your best guess at the range of values needed for the Y-axis (Ymin and Ymax). Let's use Xmin=0, Xmax=100, Ymin=-80, Ymax=40. Click OK.

(4) Next click on Numerics in the Auto window. Ntst is the number of collocation points, and you typically don't need to change it. Nmax is the maximum number of steps to take when making bifurcation curves. I usually set it at 2000. NPr sets frequency of printing a label on a bifurcation curve. Note that bifurcation points are automatically labeled, so this just applies to points

that are not bifurcations. I usually set it to 5000, which makes it so that only bifurcation points are labeled. (You can restart or continue a bifurcation diagram only from a labeled point, so using a smaller NPr would give you more potential continuation points, but I typically don't find them useful.) Ds is the initial step size. Leave it as is, unless you want to move to the left, in which case set Ds to a negative number. You won't ever need to change the next three. Dsmax is the maximum step size, which I often make smaller than the default. I'll often use Dsmax=0.2, particularly for periodic branches. (Making it smaller helps with the construction of more complicated branches.) Par Min and Par Max are the minimum and maximum values of the bifurcation parameters. They typically match what you used in the Axes window (0 and 100, in this case). When done, click OK.

(5) Finally, click on Run and Steady state. This will make a stationary bifurcation diagram, starting from the steady state that you found in the XPP window. To print the diagram click File and then Postscript. You can also save the diagram (Save diagram) and read it into AUTO later (Load diagram). You should see a solid red curve, followed by a thin black curve. The red curve is a branch of stable equilibria, while the black curve is a branch of unstable equilibria. There should be a label at the transition point (probably label 2). This labels the bifurcation. In the X-window that was used to invoke XPP you should see several rows of text and numbers. This tells us that label 2 corresponds to a Hopf bifurcation, and it occurred at about $I_{ap} = 93.86$ pA.

(6) You can move along the bifurcation diagram using Grab. Click Grab and a plus sign will appear at label 1. Notice that at the bottom of the AUTO window some information will appear, saying that the location is on branch 1, point 1, label 1, primary parameter $I_{ap} = 0$, second parameter $g_{Ca} = 4.4$, and then the "norm" which for a stationary equilibrium is just the absolute value, and then the equilibrium value of V. The last entry is the period, which is meaningless on a stationary branch. You can now move along the branch using the right arrow on your keyboard. Notice that the information along the bottom of the window changes. If you were to hit the Enter key on the keyboard, then the parameter values at that point on the curve would be fed into the parameter window of XPP, and the equilibrium values would be fed into the Initial Conditions window of XPP (so you would be at an equilibrium point). But don't hit enter Enter, instead keep moving along the branch until you get to label 2. You should see from the information on the bottom that this is a HB, a Hopf bifurcation. If you move past this onto the thin black curve then notice that the Pt label goes from being negative to being positive. This is you only indication that the equilibrium went from being stable (when

Pt is negative) to unstable (when Pt is positive). Hit Enter to get out of Grab mode, followed by reDraw to make the X go away.

(7) We know from the class lectures that there should actually be a second HB. The reason it does not show up is that we did not go far enough with I_{ap}. You can see this if you click Axes and change Xmax=100 to Xmax=300. (Then again reDraw; you often need to use reDraw if something disappears.) You can also see label 3 at the end of the branch. Click Grab, and then if you hit Tab on your keyboard you will jump from one label to the next on the branch. Jump over to label 3 and hit Enter. Now click on Numerics and increase Par max to 300, followed by Ok. Then click Run. You should see the branch extend out to the right, until $I_{ap} = 300$. The latter portion of the branch is again stable, and there is a second HB at the transition point. This is the complete **stationary branch** of the bifurcation diagram.

(8) To trace out the **periodic branch** we need to again use Grab, and hop over to label 2, the HB point. Hit Enter. Then click on Run. You will be asked what you want to do. Click on Periodic. (Extend will just extend the stationary branch, like we just did.) With any luck, you should see two curves drawn out. These are the max and min V values for the periodic branch. The blue portion represents unstable periodic solutions, while the green reflects stable periodic (tonic spiking) solutions. This branch should look very much as it did in the last lecture, and is characteristic of oscillations emerging from a pair of **subcritical Hopf bifurcations**. Where the blue curves switch to green are the two **saddle-node of periodics bifurcations**.

(9) The SNP bifurcations should be labelled as LP, and you can jump over to one of them using Grab. Do that, and then use the right or left arrow to move onto a location on the stable periodic branch. Hit Enter to save this information. Now go back to your XPP screen and type IG. You should see a tonic spiking voltage trace. Open a new XPP window (click Makewindow) and put V and w on the x- and y-axes, respectively. You should now see a close curved, which is the tonic spiking limit cycle. If you use IM (Initial Conditions Mouse) and click inside and outside of the limit cycles you can see how it attracts trajectories. To get the equilibrium in the picture type SG or SM.

(10) Back in the AUTO window click File and Save diagram. You can change the default name to something you like better (like ML.auto), then hit Ok. This will save the bifurcation diagram in the same directory as your ML.ode code.

(11) Another useful way to plot the bifurcation diagram is in terms of the period vs. parameter value. To do this, click Axes and then Period. A dialogue box opens up and you can replace Ymin with 0 and Ymax is your best guess at the maximum value of the period over the whole parameter range. Set

Ymax=200 and hit Ok. Then a bifurcation diagram of Period vs. I_{ap} should appear. What is the range of periods for the stable portion of the periodic branch?