Final Exam (Graph Theory and Networks)
April 28

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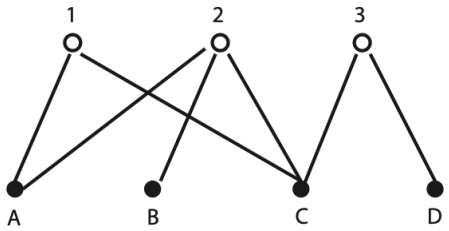
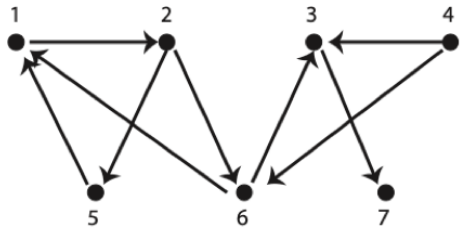
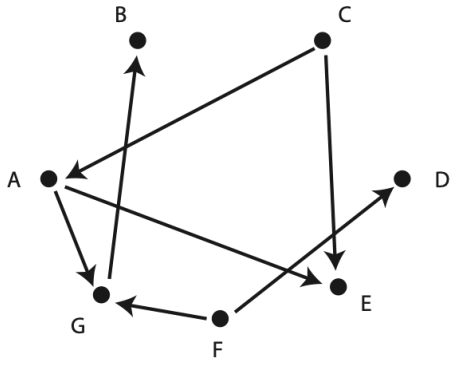
(1) (10 pts) Consider the bipartite network below, with 3 groups (top row) and 4 actors (bottom row).
   (a) Give the incidence matrix **B**.
   (b) Sketch graphs of the two weighted one-mode projections and give their adjacency matrices.



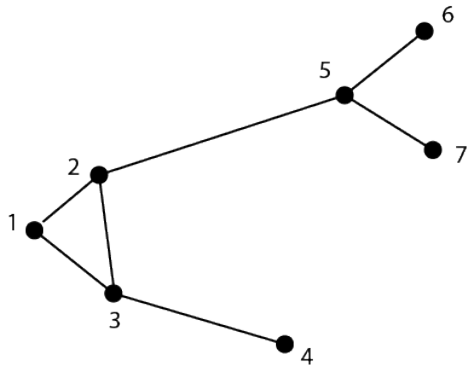
(2) (8 pts) For the directed graph below find the following:
   (a) In-component of node 6.
   (b) Out-component of node 6.
   (c) The strongly-connected component that contains node 6.
   (d) The weakly-connected component that contains node 6.



(3) (10 pts) Suppose that the directed graph below is a citation network. Sketch the cocitation and bibliographic coupling network graphs.



(4) (9 pts) Consider the undirected graph below. For each question, in case of a tie, answer with all the tied top nodes.



(a) Which node has the highest degree centrality?
(b) Which node has the highest betweenness centrality?
(c) Which node has the highest closeness centrality?

(5) (10 pts) The modularity of a cluster of nodes, C, is $L_c - \frac{k_c^2}{4L}$, where L_c is the number of internal links in C, L is the number of links in the network, and k_c is the total degree of nodes in C. Determine the modularity of a bisection of the complete graph K_5 in which one cluster contains two nodes and the other contains three nodes.

(6) (10 pts) What is the clustering coefficient for a node in a complete graph with n nodes? For a node in tree with n nodes?

(7) (8 pts) How many ways are there to form a simple undirected network with 4 nodes and 3 edges? How many ways are there to form a simple undirected

network with n nodes and m edges?

(8) (5 pts) Suppose you are making an Erdős-Rényi network with 1000 nodes and about 3000 edges. Give a value of the link probability p that could lead to this outcome.