1. Constructing phase portraits
For the following systems find steady states, use linear stability analysis to classify them, sketch nullclines, and draw a phase portrait. Also, indicate the basins of attraction of any stable steady states and, for saddle points, indicate the stable and unstable manifolds near the saddle point.

(1)
\[
\begin{align*}
\dot{x} &= x - x^3 \\
\dot{y} &= -y
\end{align*}
\]

(2)
\[
\begin{align*}
\dot{x} &= x(3 - x - y) \\
\dot{y} &= y(2 - x - y)
\end{align*}
\]

2. Another phase portrait
Consider the system \( \dot{x} = y^3 - 4x, \dot{y} = y^3 - y - 3x \).

(1) Find and classify all equilibria.
(2) Show that the line \( y = x \) is invariant, i.e., any trajectory that starts on it stays on it.
(3) Show that \( |x(t) - y(t)| \to 0 \) as \( t \to \infty \) for all other trajectories.
(4) Sketch a phase portrait.

3. Determining global stability with the Liapunov method
Construct a Liapunov function to show that
\[
\begin{align*}
\dot{x} &= -x + 2y^3 - 2y^4 \\
\dot{y} &= -x - y + xy
\end{align*}
\]
has no periodic solutions. (Hint: Try \( V = x^m + ay^n \))

4. Analysis of a gradient system
Consider
\[
\begin{align*}
\dot{x} &= y + 2xy \\
\dot{y} &= x + x^2 - y^2
\end{align*}
\]
First show that this is a gradient system and then find a potential function \( V \).