Introduction to Computational Neuroscience (Spring 2018)

Phase Plane Analysis of the Morris-Lecar Model

A huge advantage of working with a planar (i.e., two-dimensional) model of electrical excitability is that its dynamics can be analyzed in the phase plane. We will do this using the Morris-Lecar model, and explore how the dynamics change as a parameter is varied. The Morris-Lecar model is

$$\begin{aligned}\frac{dV}{dt} &= -[I_{Ca} + I_K + I_L + I_{ap}]/C_m \\ \frac{dw}{dt} &= \phi\left[\frac{w_\infty(V) - w}{\tau_w(V)}\right]\end{aligned}$$

where ϕ is a scaling factor for the rate of change of w. You can download this from my web site (under Course Software).

Exploration

(1) When you run the code you should see the voltage come to rest near -60 mV. This is with no applied current. Create a second window and put V on the x-axis and w on the y-axis (make sure you pick reasonable values for your x, y, min/max values). To get the nullclines click Nullclines and New, or type "nn" (if you want to export the nullclines you can then do Nullclines Save, which will put them into a data file). Where is the equilibrium? To see if it attracts trajectories type "im" (InitialConds Mouse). Then click anywhere on the phase plane and that will set the initial condition and start a trajectory from that point. To see what the time course of that trajectory looks like make the original window the active window and type "r". You can use "im" over and over again to view various trajectories (but remember that you must be in the phase plane to do this, not the V vs. t plane). Generate some passive responses, and some active responses. Are there some in which it is hard to tell? .

(2) Increase the applied current to 60 pA in the parameter window and generate a new set of nullclines. What happened to the V-nullcline? Why was there no effect on the w-nullcline? Use 'im" to see if the new equilibrium is stable or unstable. Another way is to look at the eigenvalues. To do this, click on Sing pts (which means Singular Points) and Go or type "sg". To do this, the phase plane must be the active window. A small window will pop up asking if you want to Print eigenvalues. Click Yes. The eigenvalues will be printed on the window from which XPP was invoked. Does the eigenvalue information

agree with your prior assessment of stability? (Rather than using Sing pts Go you could use Sing pts Mouse or type "sm" and then click on the screen somewhere near the equilibrium point.)

(3) Increase the applied current to 150 pA and generate a new set of nullclines. From the location of the equilibrium, do you expect the equilibrium to be stable or unstable? Use Sing pts to verify. (When you do this you will be asked about drawing strong sets, click No.) Now draw some trajectories. What happens? What does the behavior look like in the V vs. t plane? In the phase plane, use "im" to check on the stability of the limit cycle.

(4) Increase the applied current to 300 pA and generate a new set of nullclines. Do you expect the equilibrium to be stable or unstable. Confirm with Sing pts. Generate some trajectories. Do you see any spikes if you start at different places within the phase plane?

(5) By varying I_{ap} and looking at the nullclines find approximate parameter values where Hopf bifurcations occur.

(6) Action potentials occur because of the separation of time scales between the Ca^{2+} and K^+ currents (the former activates before the latter). By increasing the scaling factor ϕ you can speed up the activation of I_K by increasing the rate of change of w. The default value of ϕ is 0.04. Investigate what happens when you increase it in steps (say to $\phi = 0.1, 0.2, 0.3, 0.4, 0.5$) up to 0.5, and using $I_{ap} = 150$ pA. (You may want to freeze time courses and trajectories so that they don't go away when you do another simulation. To do this, use "gff".) What happens to the nullclines when you increase ϕ? Does the location of the equilibrium change" Why or why not? What happens to the spike frequency and amplitude? How does the limit cycle change? Are there any bifurcations? If so, at approximately what value of ϕ?