

**MAA 4226/5306 Advanced Calculus Hand-in Homework Bowers Fall 2003**

**Assignment:** Exercises 3, 4, 5, 7, 8, and 20 on pages 78–80 of Rudin and the following exercises.

1. If  $A$  is a subset of the metric space  $X, d$ , define the diameter of  $A$  by

$$\text{diam } A = \sup\{d(a, b) : a, b \in A\}.$$

Show that if  $A$  is compact, then there are elements  $a$  and  $b$  in  $A$  such that  $d(a, b) = \text{diam } A$ .

2. Suppose that  $\{a_n\}$  and  $\{b_n\}$  are sequences of positive numbers and that

$$\lim \frac{a_n}{b_n} = L,$$

where  $L \in (0, \infty)$ . Show that  $\sum a_n$  converges if and only if  $\sum b_n$  converges.

**Due date:** Monday, 08 December by 11:00 am.

You must write precise, complete arguments that convince me that you have a proof in order to receive full credit. You of course may be convinced that you understand a correct argument for proving a statement, but your job is only half finished once you have such an understanding. You must then convey that understanding to a second party, in this case to me. This is a skill that is learned only by practice. A guide as to whether you have a sound argument might be whether a fellow student can understand your proof. You are allowed to discuss with other students your arguments, but there should be only discussion and exchange of ideas—no copying another's work. If you are stuck at some point in an argument, or if you think you have a good idea but are not able to push it all the way to a proof, you may discuss this with me or others. Once you understand how a proof goes, you must write the proof yourself, in your own words. It will be apparent when you take the in-class exams if you haven't taken my instructions to heart.

As an aid to my grading, I would like your work organized as follows. First, write on one side of your paper with your name on the top of each sheet. Second, when you begin a new exercise, start with a fresh sheet of paper. Third, for most problems, the proof can fit on one or two pages. Don't be terse, but avoid excessive wordiness and try to fit your argument for a single exercise on one, at most two, pages. Finally, staple in the upper left hand corner or hand in unstapled, but do not use paper clips, do not hand in notebooks or spiral binders, and do not fold. These requirements will make my life easier when it comes to grading. Thanks for your attention to these technical details.