MAA 4227 ADVANCED CALCULUS II HOMEWORK 3

BOWERS

Exercise 1. Prove the following statements.

(i) If the real valued function f has a bounded derivative on the interval J, then f is uniformly continuous on J. (Note that J need not be closed nor bounded.)

(ii) If f has a bounded derivative on the bounded open interval $J = (a, b)$, then the one-sided limits $f(a+)$ and $f(b-)$ exist and are finite.

(iii) Let $f\colon (a, b) \to \mathbb{R}$ be differentiable on its domain. If the right-hand limit $\lim_{x \to a+} f'(x)$ exists, say equal to L, then the right-hand limit $f(a+)$ exists and

$$\lim_{h \to 0+} \frac{f(a+h) - f(a+)}{h} = L.$$

Exercise 2. Problem 14, page 115 of Rudin.

Exercise 3. Problem 7, page 138 of Rudin.

Exercise 4. Problem 15, page 141 of Rudin.

Exercise 5. Problem 16, page 141 of Rudin.

Date: Due: 11am–Monday–29–March–2004.