

Section 9.3 The Ellipse

Ellipse: the collection of points the sum of whose distances from two fixed points, called the foci (F_1 and F_2), is a constant.

Major axis: the line containing the foci

center: midpoint, C , of the line segment connecting the foci

Minor axis: the line perpendicular to the major axis and passing through the center

Vertices: the two points, V_1 and V_2 , of intersection of the ellipse and the major axis

The ellipse is symmetric with respect to its major axis and with respect to its minor axis.

$$\begin{aligned}d(F_1, P) &- \text{distance from } F_1 \text{ to } P \\d(F_2, P) &- \text{distance from } F_2 \text{ to } P \\d(F_1, P) + d(F_2, P) &= 2a \leftarrow a \text{ constant}\end{aligned}$$

Using the distance formula ($\sqrt{((x+c)^2 + y^2)} + \sqrt{((x-c)^2 + y^2)} = 2a$) we get:

Equation of an Ellipse

Center at $(0, 0)$; Foci at $(\pm c, 0)$; Vertices $(\pm a, 0)$; Major Axis along x-axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a > b > 0 \text{ and } b^2 = a^2 - c^2$$

Ex Find the equation of the ellipse with center at the origin, a focus at $(3, 0)$ and a vertex at $(-4, 0)$. Graph the equation.

Ex Find the center, vertices, foci, and graph $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

Equation of an Ellipse

Center at $(0, 0)$; Foci at $(0, \pm c)$; Vertices $(0, \pm a)$; Major Axis along y-axis

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1, \text{ where } a > b > 0 \text{ and } b^2 = a^2 - c^2$$

Ex Find the vertices, foci, and graph the equation of $9x^2 + y^2 = 9$

Ex Find the equation of the ellipse with vertices at (0,-3) and (0,3) and focus at (0,2).

Ellipse with Center at (h, k) and Major Axis parallel to a coordinate axis
 $(a > b \text{ and } b^2 = a^2 - c^2)$

<i>Center</i>	<i>Major Axis</i>	<i>Foci</i>	<i>Vertices</i>	<i>Equation</i>
(h, k)	parallel to x -axis	$(h \pm c, k)$	$(h \pm a, k)$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
(h, k)	parallel to y -axis	$(h, k \pm c)$	$(h, k \pm a)$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

Ex Find the equation of the ellipse with center at the $(2, -3)$, a focus at $(3, -3)$ and a vertex at $(5, -3)$. Graph the equation.

Ex Find the center, foci, and vertices of the ellipse $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{49} = 1$

Ex Find the equation of the ellipse given the following graph.