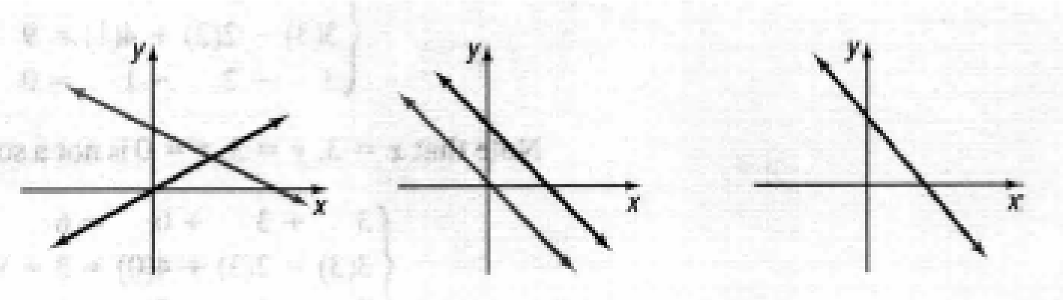
Section 10.1 Systems of Linear Equations in Two Variables

Geometric Meaning of Two Linear Equations in Two Variables

1. **Reminder of linear equation in two variables**
   A linear equation containing two variables, x and y, is an equation of the form Ax+By=C, where A, B and C. The graph of such an equation is a LINE in xy-space.
2. **A system of two linear equations in two variables.**
   Thus, the system of two linear equations containing the variables, x and y, is a pair of LINEs in xy-space. Each (x, y) pair that satisfies the system of two equations must satisfy both equations, i.e. the pair (x, y) must be on both lines.



(a) Intersecting lines; system has one solution (b) Parallel lines; system has no solution (c) Coincident lines; system has infinitely many solutions

3. **Possible solutions of linear systems**
   - Exactly ONE solution (UNIQUE solution). The solution is exactly the point where the two lines which the two equations represent intersect.
   - NO solution. This is the second case, where two lines are parallel to each other. There is no point that could be on both lines because parallel lines never intersect.
   - INFINITELY MANY solutions. This is the third case, where the two line overlaps. Essentially, this is to say that the two equations in the system are the same.

   ATTN: IN NO CASE can a linear system has exactly two solutions.
   A system of equations is said to be **consistent** if the system has AT LEAST ONE solution.
   If a system does not have a solution, the system is said to be **inconsistent**.

Solving System of Equations by Elimination

Exercise 1

[10.1.1PT]Select the type of solution for the following system

$$\begin{cases} 3x + 6y = 6 \\ -2x - 4y = -4 \end{cases}$$

○ None of these

○ Exactly two solutions

○ No solution

○ Infinitely many solutions

○ A unique solution

Equivalent Systems (Theoretical but Important)

Two systems of linear equations are **equivalent** if the two systems have identical solutions. The method of elimination is usually preferred over substitution if substitution leads to fractions or if the system contains more than two variables.

The idea behind the method of elimination is to keep replacing the original equations in the system with EQUIVALENT equations until a system of equations with an obvious solution is reached.

Let us have another look at the method of elimination we used.

Exercise 2

[10.1.1PT]Select the type of solution for the following system

$$\begin{cases} 4x - 4y = 8 \\ 3x - 3y = 6 \end{cases}$$

○ Infinitely many solutions

○ A unique solution

○ Exactly two solutions

○ None of these

○ No solution