A matrix is in **echelon form** when:

1) Each row containing a non-zero number has the number “1” appearing in the row’s first non-zero column. (Such an entry will be referred to as a “leading one”.)

2) The column numbers of the columns containing the first non-zero entries in each of the rows strictly increases from the first row to the last row. (Each leading one is to *the right* of any leading one above it.)

3) Any row which contains all zeros is below the rows which contain a non-zero entry.

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The three conditions above will ensure that the entries *below the leading ones* (in each row which contains a non-zero entry) are *all zeros*.

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A matrix is in **reduced echelon form** when:

in addition to the three conditions for a matrix to be in echelon form,

the entries *above the leading ones* (in each row which contains a non-zero entry) *are all zero’s*.

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*Note that if a matrix is in* ***Reduced Row Echelon Form*** *then it must also be in* ***Echelon form.***

To Determine if a Matrix is in **Reduced Row Echelon Form**:

<u>Circle</u> the first non-zero entry in each row of the matrix.

Then verify that:

1) any row with <u>no</u> non-zero entry is at the bottom of the matrix,

2) the circled entries are all 1's - and will be referred to as "***leading one's***",

3) each ***leading one*** is to the right of any ***leading one*** above it, and

4) all entries above and below the ***leading one's*** are zeros, that is, all other entries in the same column as a circled 1 are zeroes.

If conditions 1-4 above are satisfied, then the matrix is in **Reduced Row Echelon Form**.

If conditions 1-4 above are satisfied, with the possible modification to condition 4) that entries <u>below</u> the ***leading one's*** (but not necessarily above the ***leading one's***) are zeroes, then the matrix is in **Echelon Form**.

Examples of Matrices:

Not in Echelon Form

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

In Echelon Form, but not Reduced Row Echelon Form:

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In Reduced Row Echelon Form:

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Any matrix can be put in an equivalent **Echelon Form** using elementary row operations. Such a matrix is **<u>not</u> unique.**

For instance, the two (elementary row) equivalent matices below are both in echelon form:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} \quad R_1 = -2R_2 + R_1 \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

However, the equivalent matrix in **Reduced Row Echelon Form <u>is</u> unique**.