Solutions to requested homework problems from section 10.5 in the course notes:

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2. Given $A=\begin{bmatrix}0 & 3 & 5\\ 1 & 2 & 6\end{bmatrix}$, and $B=\begin{bmatrix}4 & 1 & 0\\ -2 & 3 & -2\end{bmatrix}$, compute $3A-2B$.

Solution:

$3A=\begin{bmatrix}0 & 9 & 15\\ 3 & 6 & 18\end{bmatrix}$ and $2B=\begin{bmatrix}8 & 2 & 0\\ -4 & 6 & -4\end{bmatrix}$. By subtracting corresponding entries, we

obtain $3A-2B$ = $\begin{bmatrix}0-8 & 9-2 & 15-0\\ 3-(-4) & 6-6 & 18-(-4)\end{bmatrix}=\begin{bmatrix}-8 & 7 & 15\\ 7 & 0 & 22\end{bmatrix}$.

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5. Given $A=\begin{bmatrix}0 & 3 & 5\\ 1 & 2 & 6\end{bmatrix}$ and $C=\begin{bmatrix}4 & 1\\ 6 & 2\\ -2 & 3\end{bmatrix}$, compute $AC-3I_2$.

Solution:

$AC=\begin{bmatrix}0 & 3 & 5\\ 1 & 2 & 6\end{bmatrix}\begin{bmatrix}4 & 1\\ 6 & 2\\ -2 & 3\end{bmatrix}=\begin{bmatrix}0+18-10 & 0+6+15\\ 4+12-12 & 1+4+18\end{bmatrix}=\begin{bmatrix}8 & 21\\ 4 & 23\end{bmatrix}$, and $3I_2=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$.

Subtracting corresponding entries, we obtain $AC-3I_2=\begin{bmatrix}8-3 & 21-0\\ 4-0 & 23-3\end{bmatrix}=\begin{bmatrix}5 & 21\\ 4 & 20\end{bmatrix}$.

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9. Find the inverse of the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 3 & 2 & -1\\ 3 & 1 & 2\end{bmatrix}$.

Solution:
(The two column method below helps to reduce careless copying or arithemtic errors.)

Sequence of row equivalent matrices **"Scratch"**

$$\left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ \mathbf{3} & 2 & -1 & 0 & 1 & 0\\ 3 & 1 & 2 & 0 & 0 & 1\end{array}\right]\xrightarrow{-3r_1+r_2=R_2}$$

$-3r_1$	-3	-3	-3	-3	0	0
r_2	3	2	-1	0	1	0
R_2	0	-1	-4	-3	1	0

Sequence of row equivalent matrices | **"Scratch"**

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ \mathbf{0} & -1 & -4 & -3 & 1 & 0 \\ \mathbf{3} & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-3r_1+r_3=R_3}$$

$-3r_1$	–3	–3	–3	–3	0	0
r_3	3	1	2	0	0	1
R_3	0	–2	–1	–3	0	1

$$\left[\begin{array}{ccc|ccc} 1 & \mathbf{1} & 1 & 1 & 0 & 0 \\ \mathbf{0} & -1 & -4 & -3 & 1 & 0 \\ \mathbf{0} & -2 & -1 & -3 & 0 & 1 \end{array}\right] \xrightarrow{r_1+r_2=R_1}$$

r_1	1	1	1	1	0	0
r_2	0	–1	–4	–3	1	0
R_1	1	0	–3	–2	1	0

$$\left[\begin{array}{ccc|ccc} 1 & \mathbf{0} & -3 & -2 & 1 & 0 \\ \mathbf{0} & -1 & -4 & -3 & 1 & 0 \\ \mathbf{0} & \mathbf{-2} & -1 & -3 & 0 & 1 \end{array}\right] \xrightarrow{-2r_2+r_3=R_3}$$

$-2r_2$	0	2	8	6	–2	0
r_3	0	–2	–1	–3	0	1
R_3	0	0	7	3	-2	1

$$\left[\begin{array}{ccc|ccc} 1 & \mathbf{0} & \mathbf{-3} & -2 & 1 & 0 \\ \mathbf{0} & -1 & -4 & -3 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & 7 & 3 & -2 & 1 \end{array}\right] \xrightarrow{3r_3+rR_1=R_1}$$

$3r_3$	0	0	21	9	–6	3
$7\,r_1$	7	0	–21	–14	7	0
R_1	7	0	0	– 5	1	3

$$\left[\begin{array}{ccc|ccc} 7 & \mathbf{0} & \mathbf{0} & -5 & 1 & 3 \\ \mathbf{0} & -1 & \mathbf{-4} & -3 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & 7 & 3 & -2 & 1 \end{array}\right] \xrightarrow{4r_3+7r_2=R_2}$$

$4r_3$	0	0	28	12	–8	4
$7r_2$	0	–7	–28	–21	7	0
R_2	0	–7	0	– 9	–1	4

$$\left[\begin{array}{ccc|ccc} 7 & \mathbf{0} & \mathbf{0} & -5 & 1 & 3 \\ \mathbf{0} & -7 & \mathbf{0} & -9 & -1 & 4 \\ \mathbf{0} & \mathbf{0} & 7 & 3 & -2 & 1 \end{array}\right] \xrightarrow[_2]{\frac{1}{7}r_1=R_1,\ -\frac{1}{7}r_2=R_2,\ \frac{1}{7}r_3=R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & \mathbf{0} & \mathbf{0} & -\frac{5}{7} & \frac{1}{7} & \frac{3}{7} \\ \mathbf{0} & 1 & \mathbf{0} & \frac{9}{7} & \frac{1}{7} & -\frac{4}{7} \\ \mathbf{0} & \mathbf{0} & 1 & \frac{3}{7} & -\frac{2}{7} & \frac{1}{7} \end{array}\right].$$

So $A^{-1} = \begin{bmatrix} -\frac{5}{7} & \frac{1}{7} & \frac{3}{7} \\ \frac{9}{7} & \frac{1}{7} & -\frac{4}{7} \\ \frac{3}{7} & -\frac{2}{7} & \frac{1}{7} \end{bmatrix}.$