Section 11.1 Sequences

Sequence Notations

Definition: A **sequence** is a list of infinitely many ordered numbers, whose indices are positive integers starting from 1.

1. **n-th term formula:** We use $\{a_n\}$ to denote the sequence, $a_1, a_2, a_3 \cdots$. The formula in the curly brackets gives the formula for n-th term.

Exercise 1

[11.1.1aPT]The first four terms of the sequence $\{(-1)^n \frac{2n-1}{n}\}$ are

○ $-1, \frac{3}{2}, -\frac{5}{3}, \frac{7}{4}$

○ $\frac{7}{4}, -\frac{5}{3}, \frac{3}{2}, -1$

○ $1, -\frac{3}{2}, \frac{5}{3}, -\frac{7}{4}$

○ $-\frac{7}{4}, \frac{5}{3}, -\frac{3}{2}, 1$

Exercise 2 (Check if formulas give the right first 4 terms)

[11.1.1cPT] The n^{th} term of the sequence $1, -\frac{1}{9}, \frac{1}{125}, -\frac{1}{2401}, \cdots$ is

○ $(-1)^n(\frac{1}{2n-1})^n$

○ $(-1)^{n+1}(\frac{1}{n})^{n+1}$

○ $(-1)^{n+1}(\frac{1}{n})^{n-1}$

○ $(-1)^{n-1}(\frac{1}{2n-1})^n$

2. **Recursive formula:** A sequence can be defined recursively, i.e., except the given first term, each term of the sequence is defined by a formula involving the previous terms.

Exercise 3

[11.1.1bPT]The first four terms of the recursive sequence given by $a_1 = 2, a_n = \frac{n}{a_{n-1}}$ are

○ $2, 1, 3, 12$

○ $2, 1, 3, \frac{4}{3}$

○ $2, \frac{1}{2}, 6, \frac{2}{3}$

○ None of these

Summation Notation

- $\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$
- $\sum_{k=1}^{\infty} a_k = a_1 + a_2 + \cdots$

Note: The index here for terms is k, called the **index of summation**. Unlike the index of a sequence, the k may not start from 1 and may end at a finite number.

Exercise 4

[11.1.2aPT]$\sum_{k=1}^{4}(k^2 - 3) =$

○ 11
○ 13
○ 27
○ 18

Exercise 4

[11.1.2bPT]Select the summation notation for $\frac{2}{e^2} - \frac{3}{e^3} + \cdots + \frac{10}{e^{10}}$

○ None of these

○ $\sum_{k=1}^{9} \frac{(-1)^k k}{e^k}$

○ $\sum_{k=0}^{8} \frac{(-1)^k (k+1)}{e^{k+1}}$

○ $\sum_{k=1}^{9} \frac{(-1)^{k+1} (k+1)}{e^{k+1}}$