Section 11.3 Geometric Sequences

Geometric Sequence

Definition: A **geometric sequence** is a sequence $\{a_n\}$, which satisfies the following recursive deifinition:

- $a_1 = a$
- $a_n = ra_{n-1}$

So the sequence is essentially determined by two parameters: the first tem, a, and the **common ratio**, r.

n-th Term Formula

Given the first term a_1 and the common ratio r, the formula for the n-th term is:

$a_n = ar^{(n-1)}$

i.e.

$a_1 = ar^{(1-1)} = a$

$a_2 = ar^{(2-1)} = ar$

$a_3 = ar^2$

Exercise 1

[11.3.1aPT]The 7^{th} term of a geometric sequence with first term $a_1 = \sqrt{2}$ and common ratio $r = -\sqrt{2}$ is

○ $8\sqrt{2}$

○ $-4\sqrt{2}$

○ $-2\sqrt{2}$

○ 4

Exercise 2

[11.3.1bPT]The n^{th} term of a geometric sequence with first term $a_1 = 2$ and common ratio $r = -\frac{1}{3}$ is

○ None of these

○ $(-\frac{2}{3})^{n-1}$

○ $2(-\frac{1}{3})^{n-1}$

○ 6^{n-1}

○ $-2(\frac{1}{3})^{n-1}$

○ $-\frac{1}{3}(2)^{n-1}$

Exercise 3

[11.3.2aPT]If a geometric sequence has $a_{46} = 12$ and $a_{47} = 13$, what is the common ratio?

○ None of these

○ $\frac{46}{47}$

○ $\frac{12}{13}$

○ $\frac{47}{46}$

○ $\frac{13}{12}$

Sum of finite terms and infinite terms of a geometric sequence

Finite Sum

- $S_n = a_1 + a_2 + a_3 + \cdots a_n = a + ar + ar^2 + ar^3 + \cdots ar^{n-1} = a\frac{1-r^n}{1-r}$
- $S_\infty = a_1 + a_2 + a_3 + \cdots = a + ar + ar^2 + ar^3 + \cdots = a\frac{1}{1-r}$, **if |r|<1**. Otherwise, the sum of all terms is undefined.

Exercise 4

[11.3.3bPT]Find the sum of the infinite geometric series $\frac{1}{25} + \frac{1}{125} + \cdots + \frac{1}{5^{n+1}} + \cdots$

○ $\frac{1}{20}$

○ $\frac{1}{12}$

○ $\frac{1}{10}$

○ $\frac{1}{15}$

Exercise 5

[11.3.3cPT]Find the sum of the alternating infinite geometric series $\frac{1}{16} - \frac{1}{64} + \cdots + (-1)^{n+1}\frac{1}{4^{n+1}} + \cdots$

○ $\frac{1}{30}$

○ $\frac{1}{6}$

○ $\frac{1}{20}$

○ $\frac{1}{12}$

Writing Repeating Decimal as a Fraction

Example. Repeating Decimal Write $0.898989\cdots$ as a fraction.

Solution. $0.898989\cdots = \frac{89}{100} + \frac{89}{100^2} + \frac{89}{100^3} + \cdots$

$$= \frac{89}{100}\left[1 + \frac{1}{100} + \frac{1}{100^2} + \cdots\right]$$

$$= \frac{89}{100}\frac{1}{1-\frac{1}{100}}$$

$$= \frac{89}{99}$$

Exercise 6

[11.3.3dPT]If the repeating decimal $0.387387387\cdots$ is written as $\frac{m}{n}$ in reduced form where m and n are integers, then $m =$

○ 11

○ 387

○ 29

○ 43

Exercise 7

[11.3.3ePT]If the repeating decimal $1.424242\cdots$ is written as $\frac{m}{n}$ in reduced form where m and n are integers, then $m =$

○ 14
○ 15
○ 47
○ 42