Section 11.5 Binomial Theorem

Binomial Coefficients

Definition:

$$C_j^n = \binom{n}{j} = \frac{n!}{j!(n-j)!},$$

where $n! = n\cdot(n-1)!$ (recursive) or $n! = n\cdot(n-1)\cdot(n-2)\cdot\cdots\cdot 3\cdot 2\cdot 1$ (direct).

Notice: 1!=1 and 0!=1 (This is the definition.)

Remark: The above notation is read as “n taken j at a time” or “n choose j”. The number gives the different possible combination one could have if he wants to choose j items from n different items. For example, if you have 52 different cards, you want to draw 5 (a hand) from them. Then you could have $C_5^{52} = \binom{52}{5} = \frac{52!}{5!47!} = 62{,}375{,}040$ distinct possible combinations. You can use this method to calculate your chance of winning a lottery. What’s your chance to become a millionaire overnight?

Binomial Theorem

- $$(a+b)^n = \binom{n}{0}a^{n-0}b^0 + \binom{n}{1}a^{n-1}b^1 + \cdots + \binom{n}{j}a^{n-j}b^j + \cdots + \binom{n}{n}a^{n-n}b^n$$

$$= \binom{n}{0}a^n 1 + \binom{n}{1}a^{n-1}b^1 + \cdots + \binom{n}{j}a^{n-j}b^j + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}1b^n$$

$$= \sum_{j=0}^{n}\binom{n}{j}a^{n-j}b^j$$

Notice: The operation in the parenthesis must be regarded as summation even if we want to expand $(a-b)^n$

Exercise 1

[11.5.2aPT]Evaluate the binary coefficient $\binom{8}{6}$

○ 28
○ 56
○ 1
○ 0

○ 8

Exercise 2

[11.5.2aPT]Evaluate the binary coefficient $\binom{5}{5}$

○ 0
○ 5
○ 120
○ 1
○ 10

Exercise 3

[11.5.1aPT]Find the coefficient of x^6 in the expansion of $(x-2)^{11}$

○ $-\frac{11!}{6!}2^5$

○ $\frac{10!}{6!4!}2^{11}$

○ $-\frac{11!}{6!5!}2^5$

○ $\frac{11!}{6!5!}2^5$

○ $\frac{11!}{5!}2^5$

Exercise 4

[11.5.1aPT]Find the coefficient of x^4 in the expansion of $(3-x)^{10}$

○ $\frac{10!}{6!}3^6$

○ $\frac{10!}{4!6!}3^6$

○ $-\frac{10!}{4!6!}3^6$

○ $-\frac{10!}{4!}3^6$

○ $-\frac{11!}{4!7!}3^{10}$