Section 3.5 Complex Numbers

(EXPONENTS AND RADICALS REQUIRED)

Definition

1. $i = \sqrt{-1}$,

   i is the building block of complex numbers. It handles the difficulty we usually have to take a square root of a negative real number. i could be considered the "same" as x in the expressions. However, i does have the following properties.

   - $i^2 = -1$
   - $i^3 = i^2 i = -i$
   - $i^4 = i^3 i = -i^2 = -(-1) = 1$; or $i^4 = (i^2)^2 = (-1)^2 = 1$
   - $i^5 = i^4 i = i$

   Remark: The powers of I repeat with EVERY 4-th power.

   Example 1

   [3.5.1aPT] Write $2i^{20} - i^{17}$ in $a + bi$ form.

   ○ $2 - i$
   ○ $2 + i$
   ○ $-2 - i$
   ○ $-i$
   ○ $-2 + i$

2. **Complex number** (**standard form**): a+bi, where a and b are real numbers. a is called the real part. B is called the imaginary part. Treating i as x, the algebraic operations for real numbers carry over:

   - **Equality:** a+bi = c+di ⇔ a=c and b=d.
   - **Addition:** (a+bi)+(c+di) ⇔ (a+c)+(b+d)i.
   - **Subtration:** (a+bi)-(c+di) ⇔ (a-c)+(b-d)i.
   - **Multiplication:** (a+bi)(c+di)=ac+adi+bci-bd=(ac-bd)+(ad+bc)i.

   The following operations are different from real numbers:

   - **Conjugate:** if z=a+bi, then the conjugate of z is a-bi, denoted by $\bar{z}$. (Conjugate is to negate the imaginary part.)

     $z + \bar{z} = 2a$
     $z - \bar{z} = 2bi$

     $z\bar{z} = a^2 + b^2$, this trick is important in doing complex division. Actually we can understand this as

$\overline{\overline{z}} = z$

$\overline{z+w} = \overline{z} + \overline{w}$

$\overline{z \cdot w} = \overline{z} \cdot \overline{w}$

Extra Credit

Show that the above 6 identities are true. Can we interpret the identity $z\overline{z} = a^2 + b^2$ by using the difference of squares formula?

- **Division:** if z=a+bi and w=c+di, then $\frac{w}{z} = \frac{w\overline{z}}{z\overline{z}} = \frac{w\overline{z}}{a^2+b^2}$

Example 2

[3.5.1bPT]Write $\frac{2-i}{1+3i}$ in the form $a+bi$

○ $\frac{1}{2} - \frac{7}{10}i$

○ $-\frac{5}{8} + \frac{7}{8}i$

○ $-\frac{1}{10} - \frac{7}{10}i$

○ $-\frac{1}{10} + \frac{7}{10}i$

3. Solving Quadratic Equations with NEGATIVE Discriminants
Recall:

Δ= $b^2 - 4ac$ >0: TWO x-intercepts

Δ= $b^2 - 4ac$ =0: ONE x-intercept

Δ= $b^2 - 4ac$ <0: NO x-intercept (i.e. no real solutions). However, after we have introduced complex numbers, we know that, by the quadratic formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, the quadratic function with real coefficients, in this case, will have a PAIR of CONJUGATE complex numbers as two solutions. (THEY ARE NOT X-INTERCEPTS THOUGH.)

Remark: How to take square root of a negative number: $\sqrt{-16} = \sqrt{-1}\sqrt{16} = 4i$.

Example 3

[3.5.2aPT] Select the real and complex zeros of $f(x) = 2x^2 - 6x + 5$

○ $3/2 + (1/2)i, 3/2 - (1/2)i$

○ none of these

○ $-3/2 + (1/2)i, -3/2 - (1/2)i$

○ $3/2, -3$