Section 3.6 Complex Zeros of REAL Polynomials

(Must understand 3.4 and 3.5 first)

Recollection of 3.4 and 3.5

Section 3.4: Every real polynomial (i.e. polynomial with real coefficients) can be completely factored into factors that are either linear factors (of degree 1) or irreducible quadratic factors (of degree 2). (An **irreducible** quadratic factor is a quadratic that cannot be factored into REAL linear factors, i.e. its discriminant is NEGATIVE.)
Section 3.5: Every real quadratic function whose discriminant is NEGAGIVE (i.e. irreducible) has a pair of conjugate zeros. (TWO complex zeros that are conjugates of each other.)

Then what can we say about COMPLEX ZEROS of a polynomial with real coefficients?

1. The complex zeros of a REAL polynomial come from those irreducible quadratic factors. And they show up in pairs. For each pair, the two complex numbers are conjugate of the other.
2. If a REAL polynomial has a complex zero, then the conjugate of this zero must also be a zero of this polynomial. (A consequence of 1.)

Exercise 1
[3.6.1aPT]Select the choice with ALL the remaining zeros of a polynomial with real coefficients and degree 5 having zeros of 3, $4-3i$, $-2+5i$.

○ $4-3i,\ -2+5i$

○ $4+3i,\ -2-5i$

○ none of these

○ $-4-3i,\ 2+5i$

○ $-4+3i,\ 2-5i$

3. A REAL polynomial of degree n must have n zeros if complex numbers are allowed. Thus, it could be factored into n linear factors. (If you see a linear factor like (x-z) or (x-a-bi), where z is a complex number (NOT real), then the factored form must have another linear factor, $(x-\bar{z})$ or (x-a+bi).) (A corollary of 1 and 2.)

Attention: Pay attention to the signs.
Exercise 2
[3.6.1bPT]Select the polynomial with real coefficients of degree 5 having zeros of 5, $-1+i$, $2+i$.

- ○ $(x-5)(x+1+i)(x+1-i)(x+2-i)(x+2+i)$
- ○ $(x-5)(x+1+i)(x+1-i)(x-2-i)(x-2+i)$
- ○ None of these
- ○ $(x-5)(x-1+i)(x-1-i)(x+2-i)(x+2+i)$
- ○ $(x-5)(x-1+i)(x-1-i)(x-2-i)(x-2+i)$