Section 3.8 Polynomial and Rational Inequalities

(Must understand 3.3)

Solving Polynomial Inequalities by Graphing

Recall: To graph the polynomial

1. Ends behavior;
2. Zeros and their multiplicity. (If the polynomial is not fully factored we need to factor it first.)
3. Determine local behavior near zeros: CROSS (EVEN multiplicity) or TOUCH (ODD multiplicity)
4. Connect the graph:

To solve the corresponding polynomial inequality, we need an extra step

5. Observe the graph and find out the intervals on which the inequality is satisfied.

Exercise 1

[3.8.1aPT]Solve $3(2x+3)(x-2)(3-x) < 0$

○ none of these

○ $(-\infty, -\frac{3}{2}) \cup (0, 2) \cup (3, \infty)$

○ $(-\frac{3}{2}, 2) \cup (3, \infty)$

○ $(-\frac{3}{2}, 0) \cup (2, 3)$

○ $(-\infty, -\frac{3}{2}) \cup (2, 3)$

Exercise 2

○ [3.8.1BPT]Solve $2x^2 - x \leq 3$

○ $(-\infty, -1] \cup [\frac{3}{2}, \infty)$

○ $[-\frac{3}{2}, 1]$

○ None of these

○ $(-\infty, -\frac{3}{2}] \cup [1, \infty)$

○ $[-1, \frac{3}{2}]$

$f(x) = \frac{p(x)}{q(x)}$ **and** $f(x) = p(x)q(x)$

	$f(x) = \frac{p(x)}{q(x)}$	$f(x) = p(x)q(x)$
At zeros of p(x) (that are not zeros of q(x))	0	0
At zeros of q(x) (that are not zeros of p(x))	undefined	0
$p(x) = +$ and $q(x) = +$	+	+
$p(x) = -$ and $q(x) = -$	+	+
$p(x) = +$ and $q(x) = -$	$-$	$-$
$p(x) = -$ and $q(x) = +$	$-$	$-$

Conclusion: The rational function $f(x) = \frac{p(x)}{q(x)}$ and the polynomial $f(x) = p(x)q(x)$ assume the same sign where both are well-defined.

Notice: Although they assume the same sign wherever defined, their graph are NOT the same. The ration function blows up and forms asymptotes near the zeros of q(x).

Then the steps for solving rational inequalities are:

1. Transform the rational inequality to equivalent polynomial inequality.
2. Solve the polynomial inequality by graphing.
3. EXCLUDE the zeros of q(x) from what we obtained from step 2 because the rational function is undefined at these points.

Example 2

[3.8.2aPT]Solve $\frac{x-5}{x+2} \geq 0$

○ None of these

○ $[5, \infty)$

○ $(-\infty, -2) \cup (5, \infty)$

○ $(-2, 5]$

○ $(-\infty, -2) \cup [5, \infty)$

Example 3

[3.8.2aPT]Solve $\frac{x(x^2+1)(x-2)}{(x-1)(x+1)} > 0$

○ None of these

○ $(0, 2)$

○ $(-\infty, -1) \cup (0, 1) \cup (2, \infty)$

○ $(-\infty, 0) \cup (2, \infty)$

○ $(-1, 0) \cup (1, 2)$