Section 3.8 - Steps in Solving Polynomial and Rational Inequalities

STEP 1 : Write the inequality so that a polynomial or rational expression *f* is on the left side and zero is on the right side in one of the following forms:

$f(x) > 0$ $f(x) \geq 0$ $f(x) < 0$ $f(x) \leq 0$

For rational expressions, be sure that the left side is written as a single quotient. This step converts the problem of solving an inequality into an **equivalent** (i.e., the same solution) problem of determining where a function is positive (or negative).

STEP 2 : Factor f(x) to determine the numbers at which the expression *f* (x) on the left side equals **zero** and, if the expression is rational, the numbers at which the expression *f* on the left side is **undefined**.
We will call these numbers ***partition values.***
The **Intermediate Value Theorem** tells us that a continuous function (graph can be drawn without raising the pencil from the paper) **cannot change signs** on an interval without having a **zero** in that interval. So the above ***partition values*** divide the x-axis into intervals on which the sign of f(x) CANNOT change.

STEP 3 : Use the numbers found in STEP 2 to separate the real number line into intervals.
We will construct a **sign chart** for the function f(x).

STEP 4 : Determine the sign of each factor of f(x) on the intervals found in step 3.

STEP 5: Multiply the sign of the factors to determine the sign of f(x) in each interval.

STEP 6:
The solution of the inequality includes all intervals with the correct sign (positive or negative).

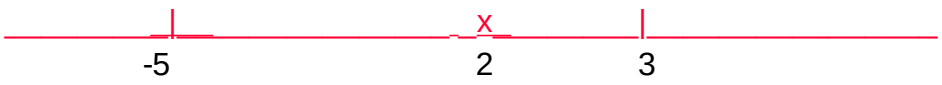
If the inequality is not strict, include the numbers at which *f (x) is zero* in the solution set.

Be careful: *The numbers which make f(x) undefined (i.e., the zeroes of the denominator in a rational function) are never included in the solution set.*

Example:

Solve $\dfrac{(3-x)(x+5)^2}{x-2} \le 0$.

We must determine for which real values of x the above rational function is less than or equal to zero. Since the function is already factored, we are ready to set up a sign chart.



(3 - x)	+ + + + + + + + + + + + + + + + + 0 - - - - - - - - -
$(x+5)^2$	+ + + + 0 +
(x - 2)	- - - - - - - - - - - - - - - - - 0 + + + + + + + + + +

Note 1: The factor (3 - x) is <u>negative</u> to the right of its zero, that is, for x > 3.
The factor (x - 2) is <u>positive</u> to the right of its zero, that is, for x > 2.
The factor $(x+5)^2$ is <u>positive</u> to the right <u>AND</u> left of its zero, that is, for all $x \neq -5$ since the square of any non-zero real number is positive.

Note 2: Zeros of the numerator are included in the solution of a non-strict inequality.

Zeros of the denominator are not included in the solution since dividing by zero is not allowed/not defined.

ans. <u>$(-\infty, 2) \cup [3, \infty)$</u>