

## Section 4.2 Exponential Functions

**(ALERT: Check all files in this folder. MASTERY of TRANSFORMATION and EXPONENTS IS ESSENTIAL.)**

### Exponential Functions.

**1. Definition:** An exponential function assume the form  $f(x) = a^x$ ,  $a > 0, a \neq 1$ . Its domain is the set of ALL real numbers. Its range is the set of all positive real numbers.

**Remark:** What is the difference between  $y = a^x$  and  $y = x^a$ ? Which is exponential function? Which is power function?

#### **2. Properties of exponentials (a>0,b>0)**

- $a^0 = 1$
- $a^{s+t} = a^s \cdot a^t$
- $(a^s)^t = a^{s \cdot t}$
- $(ab)^s = a^s b^s$
- $a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s$

## The Graph of $f(x) = a^x, a > 1$

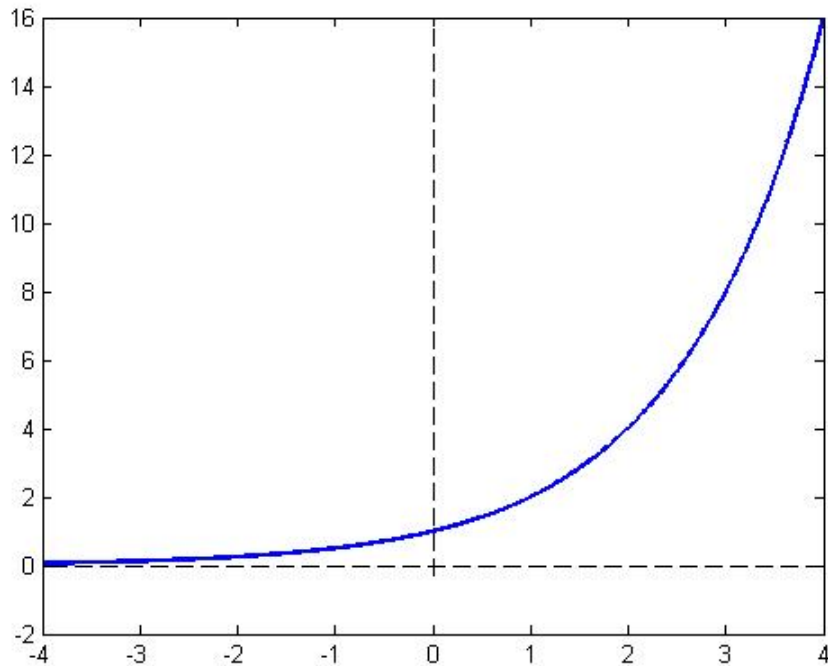


Figure 1 Graph of  $f(x) = 2^x$

**Properties:** The graph of  $f(x) = a^x, a > 1$  looks similar to  $f(x) = 2^x$

- Strictly **increasing**, i.e., if  $x_1 > x_2$ , then  $a^{x_1} > a^{x_2}$ .
- $y \rightarrow \infty$  as  $x \rightarrow \infty$
- $y \rightarrow 0$  as  $x \rightarrow -\infty$ , i.e.  $y=0$  is a horizontal asymptote.
- The y-intercept is 1.
- The graph contains the points  $(0,1)$  and  $(1,a)$ .

## The Graph of $f(x) = a^x, 0 < a < 1$

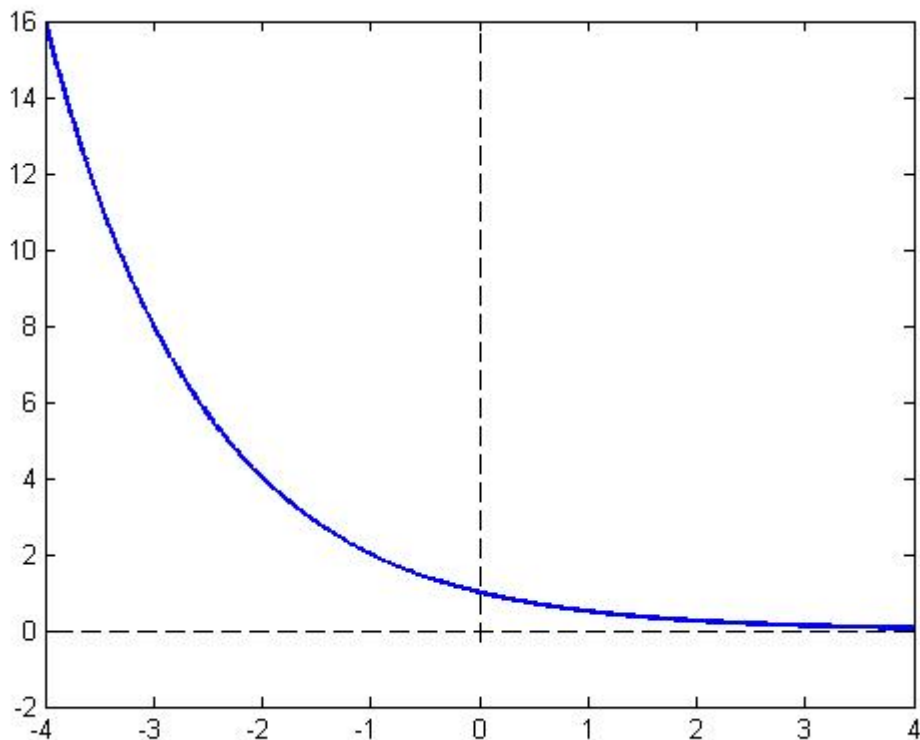


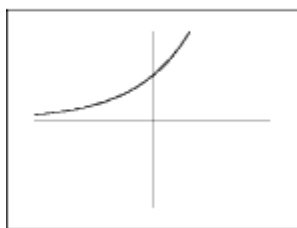
Figure 2 Graph of  $f(x) = \left(\frac{1}{2}\right)^x$

**Properties:** The graph of  $f(x) = a^x, 0 < a < 1$  looks similar to  $f(x) = \left(\frac{1}{2}\right)^x$

- Strictly **decreasing**, i.e., if  $x_1 > x_2$ , then  $a^{x_1} < a^{x_2}$ .
- $y \rightarrow \infty$  as  $x \rightarrow -\infty$
- $y \rightarrow 0$  as  $x \rightarrow +\infty$ , i.e.  $y=0$  is a horizontal asymptote.
- The y-intercept is 1.
- The graph contains the points  $(0,1)$  and  $(1,a)$ .

### Exercise 1

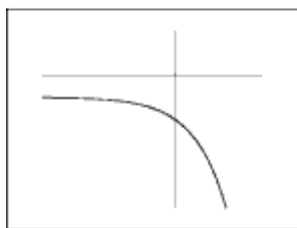
[4.2.1aPT] Select the equation of the following graph.



- $y = b^x, b > 1$
- $y = -b^{-x}, b > 1$
- $y = -b^x, b > 1$
- $y = b^{-x}, b > 1$

**Example 2**

[4.2.1cPT] Select the equation of the following graph.



- $y = -(1 + a^x), 0 < a < 1$
- $y = a^{-(1+x)}, 1 < a$
- $y = a^{-(1+x)}, 0 < a < 1$
- $y = -(1 + a^x), 1 < a$

**Exercise 3**

[4.2.1dPT] Select the graph of  $y = 1 - a^x, 1 < a$ .



