Section 4.3 Logarithmic Functions

(ALERT: Based on 4.1 4.2)

Logarithms

1. Definition: $y = \log_a(x)$ (read as y is logarithm to the base a of x.) $\Leftrightarrow x = a^y$ for all $a > 0, a \neq 1$. Plugging the first to the second, we have $x = a^{\log_a(x)}$. In another word, intuitively $\log_a(x)$ gives the power a needs to be raise to to obtain x.

- Basic properties:

  $\log_a 1 = 0$, because $a^0 = 1$,

  $\log_a a = 1$, because $a^1 = a$,

  $\log_a(a^m) = m$, because $a^m = a^m$,

  $a^{\log_a(x)} = x$, **by definition (subtle but most important).**

- Common log:

  $\log x = \log_{10} x$

- Natural log:

  $\ln x = \log_e x$, where $e = 2.71828182845904523536...$.

Example 1

[4.3.1aPT]If $(\frac{1}{2})^a = 4$, then

○ $\log_a \frac{1}{2} = 4$

○ $\log_a 4 = \frac{1}{2}$

○ $\log_{\frac{1}{2}} 4 = a$

○ $\log_4 a = \frac{1}{2}$

○ $\log_{\frac{1}{2}} a = 4$

○ $\log_4 \frac{1}{2} = a$

Example 2

[4.3.1cPT]$\log_{\sqrt{4}}(\sqrt{4})^5 =$

○ $\sqrt{4}$

○ $(\sqrt{4})^5$

○ $\frac{1}{5}$

○ $\frac{5}{2}$

○ 5

Example 3

[4.3.1dPT]$\ln \frac{1}{e^{-a}} =$

○ $-a$

○ $-\frac{1}{a}$

○ $\frac{1}{a}$

○ $\frac{1}{e^{-a}}$

○ a

Logarithmic Function and Exponential function

We know that $y = \log_a(x) \Leftrightarrow x = a^y$, so if $f(x) = \log_a(x)$ then $g(x) = a^x$ is the inverse of $f(x)$, i.e. $f^{-1}(x) = g(x) = a^x$ That is to say that the logarithmic function and the exponential function with the same base are the inverse function of each other. Then we recall from Section 4.2 the relation between a function and its inverse.

- The Domain of $y = a^x$, $(-\infty, +\infty)$, is the Range of $y = \log_a(x)$.
- The Range of $y = a^x$, $(0, +\infty)$, is the Domain of $y = \log_a(x)$.
- The two identities are more sensible if you view them as the composition of a function and its inverse: $\log_a(a^m) = m$, $a^{\log_a(x)} = x$

Exercise 4

[4.3.1ePT]The domain of $f(x) = \ln(-3x - 1)$ is

○ $(0, \infty)$

○ $(-\infty, -\frac{1}{3})$

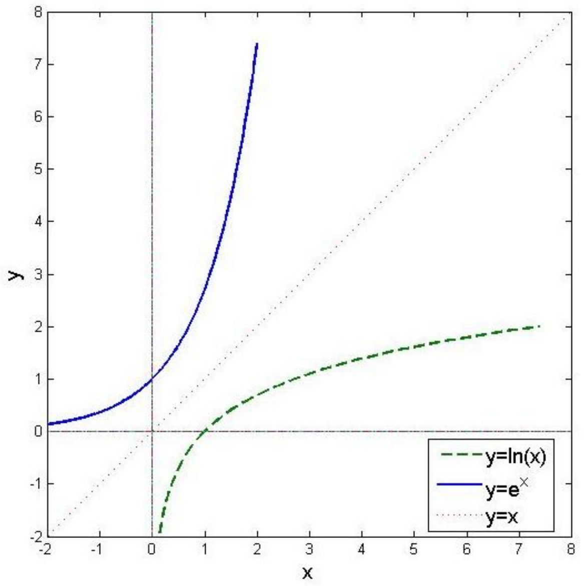
○ $[-\frac{1}{3}, \infty)$

○ $(-\infty, -\frac{1}{3}]$

○ $(-\frac{1}{3}, \infty)$

- The graph of $y = a^x$ and $y = \log_a(x)$ are symmetric about the line y=x, for any $a > 1, a \neq 1$.

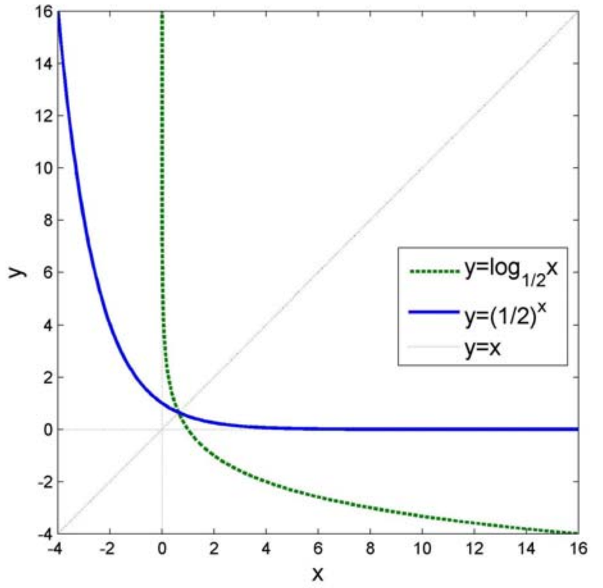
  $y = \ln(x)$ and $y = e^x$ are the inverse function of each other.



Properties: The graph of $f(x) = \log_a x, a > 1$ **looks similar to** $f(x) = \ln(x)$

- Strictly **increasing,** i.e., if $x_1 > x_2$**,** then $a^{x_1} > a^{x_2}$.
- $y \to \infty$ as $x \to \infty$
- $y \to -\infty$ as $x \to 0$, i.e. x=0 is a vertical asymptote.
- The x-intercept is 1.
- The graph contains the points (1,0) and (a,1).
- $y = \ln(x)$ and $y = e^x$ are the inverse function of each other.

$y = \log_{1/2}(x)$ and $y = \left(\frac{1}{2}\right)^x$ are the inverse function of each other.

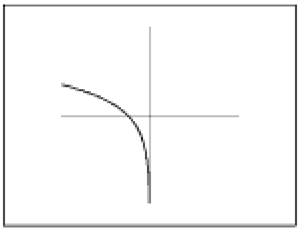


Properties: The graph of $f(x) = \log_a x, 0 < a < 1/2$ **looks similar to** $f(x) = \log_{1/2}(x)$

- Strictly **decreasing,** i.e., if $x_1 > x_2$, then $a^{x_1} < a^{x_2}$.
- $y \to \infty$ as $x \to 0$, i.e. y=0 is a vertical asymptote.
- $y \to -\infty$ as $x \to \infty$.
- The x-intercept is 1.
- The graph contains the points (1,0) and (a,1).
- $y = \log_{1/2}(x)$ and $y = \left(\frac{1}{2}\right)^x$ are the inverse function of each other.

Exercise 5

[4.3.2aMSPT]Select ALL the correct equations for the given graph.



□ $y = \log_a(-x), 0 < a < 1$

□ None of these

□ $y = \log_a(-x), a > 1$

- ☐ $y = -\log_a(-x), 0 < a < 1$
- ☐ $y = -\log_a(-x), a > 1$