

Section 4.4 Properties of Logarithms (Based on 4.2 & 4.3)

Repeated Properties

- $y = \log_a(x) \Leftrightarrow x = a^y$ for all $a > 0, a \neq 1$.
- $\log_a 1 = 0$.
- $\log_a a = 1$, because $a^1 = a$.
- $\log_a M^r = r \cdot \log_a M$.
- $\log_a a^r = r$.
- $a^{\log_a x} = x$.
- $M > 0, N > 0$ and $\log_a M = \log_a N \Rightarrow M = N$.
- $a^x = a^y \Rightarrow x = y$.
- $\log_a MN = \log_a M + \log_a N$.
- $\log_a \frac{M}{N} = \log_a M - \log_a N$.

Exercise 1

[4.4.1aPT] If $\log_4 2 = a$ and $\log_4 3 = b$, then $\log_4 54 =$

- $3ab$
- $3(a + b)$
- $a + b^3$
- ab^3
- $a + 3b$

Exercise 2

[4.4.1bPT] Expand $\ln\left(\frac{\sqrt{x}}{(y-3)^{-2}z^4}\right)$

- $\frac{1}{2} \ln x - 2 \ln (y - 3) + 4 \ln z$
- $\frac{1}{2} \ln x + 2 \ln (y - 3) - 4 \ln z$
- $\frac{1}{2} \ln x + 2 \ln y - 2 \ln 3 - 4 \ln z$
- $\frac{1}{2} \ln x - 2 \ln (y - 3) - 4 \ln z$

Exercise 3

[4.4.1cPT] $8 \log y - \frac{1}{5} \log x - 2 \log z =$

- $\log \left(\frac{y^8 z^2}{\sqrt[5]{x}} \right)$
- $\log \left(\frac{y^8 \sqrt[5]{x}}{z^2} \right)$
- $\log \left(\frac{y^8}{\sqrt[5]{x} z^2} \right)$
- $\log \left(\frac{\sqrt[5]{x} z^2}{y^8} \right)$

Exercise 4

[4.3.1dPT] $\ln \frac{1}{e^{-a}} =$

- $-a$
- $-\frac{1}{a}$
- $\frac{1}{a}$
- $\frac{1}{e^{-a}}$
- a

Exercise 5

[4.4.2bPT] $e^{(5 \ln 2 - \ln 3)} =$

- -5
- $\frac{25}{3}$
- None of these
- $\frac{32}{3}$
- 22

Exercise 6

[4.4.2cPT] If $\ln y = 4 \ln x + \ln 2$, then $y =$

- $2x^4$
- e^{x^4+2}
- e^{2x^4}
- $x^4 + 2$
- $4x + 2$

Exercise 7

[4.4.2dPT] If $\ln y = 5x - 2 \ln 3$, then $y =$

- $\frac{e^{5x}}{9}$
- $e^{\frac{5x}{9}}$
- e^{5x-9}
- $e^{5x} - 9$
- $-9e^{5x}$

Change of Base Formula

If $a > 0, a \neq 1, b > 0, b \neq 1$ and $M > 0$, then $\log_a M = \frac{\log_b M}{\log_b a}$ (change from base a

to base b). Consequently

- $\log_a M = \frac{\log M}{\log a}$, base 10, i.e. common log
- $\log_a M = \frac{\ln M}{\ln a}$, base e , i.e. natural log

Exercise 8

[4.4.1dMSPT] Select ALL of the correct formulas/statements if $x > 0, y > 0, x \neq 1, y \neq 1$

- $\log \frac{x}{y} = \log x - \log y$

- $\log_y y = 1$
- $\log_5 4 = \frac{\log 5}{\log 4}$
- $\log_{-5}(-5)$ not defined
- $\log xy = \log x \log y$
- $\log_2 \frac{1}{2} = -1$

Exercise 9

[4.4.1dPT] Select the correct formula or statement if $x > 0, y > 0, x \neq 1, y \neq 1$

- $\frac{\log x}{\log y} = \log \frac{x}{y}$
- $\log_7 10 = \frac{10}{7}$
- $\log_3 3^{-5}$ is not defined
- $\log_{\frac{1}{4}} \frac{1}{4} = 4$
- $\log_x 1 = 1$
- $\log xy = \log x + \log y$