

Section 4.5 Logarithmic and Exponential Equations (Based on 4.2, 4.3 & 4.4)

Repeated Properties

- $y = \log_a(x) \Leftrightarrow x = a^y$ for all $a > 0, a \neq 1$.
- $\log_a 1 = 0$.
- $\log_a a = 1$, because $a^1 = a$.
- $\log_a M^r = r \cdot \log_a M$.
- $\log_a a^r = r$.
- $a^{\log_a x} = x$.
- $M > 0, N > 0$ and $\log_a M = \log_a N \Rightarrow M = N$.
- $a^x = a^y \Rightarrow x = y$.
- $\log_a MN = \log_a M + \log_a N$.
- $\log_a \frac{M}{N} = \log_a M - \log_a N$.

Logarithmic and Exponential Equations.

The basic idea is to unify the bases of all terms on both sides.

Exercise 1

[4.5.1aPT] Find x if $\log_4\left(\frac{1}{16}\right)^x = -3$.

- $-\frac{3}{2}$
- $-\frac{2}{3}$
- $\frac{2}{3}$
- $\frac{3}{2}$

Exercise 2

[4.5.1bPT] Find x if $\log_x \frac{1}{16} = -2$.

- No solution
- $-4, 4$
- 4
- $\frac{1}{4}$

Exercise 3

[4.5.1cPT] Select the choice with all x such that $\log_2(3x - 2) - \log_2(x + 1) = 3$.

- 3
- No solution
- -2
- 2
- $\frac{17}{23}$

Exercise 4

[4.5.1fPT] Select the choice containing all x such that $\frac{1}{3} \log_4 x = \log_4 \frac{1}{3}$.

- 27
- $\frac{1}{\sqrt[3]{3}}$
- $\frac{1}{27}$
- None of these
- 1

Exercise 5

[4.5.2aPT] Find x such that $4^{(x+1)} = 8$.

- 1
- $\frac{7}{4}$
- $\frac{1}{2}$
- None of these

Exercise 6

[4.5.2bPT] Find all x such that $e^{-3-2x} = \frac{1}{e^2}$.

- $-\frac{5}{2}$
- $-\frac{7}{4}$
- None of these
- $-\frac{1}{2}$
- $\frac{1}{2}$

Exercise 7

[4.5.3aPT] Find all x such that $4^{(2x)} = 3$.

- None of these
- $\ln \frac{3}{8}$
- $\ln \frac{3}{16}$
- $\frac{\ln 3}{\ln 8}$
- $\frac{\ln 3}{\ln 16}$

Exercise 8

[4.5.3bPT] Find all x such that $e^{(4-3x)} = 3$.

- $\frac{1}{3}$
- $\frac{\ln 3}{3} - 4$
- $\frac{4+\ln 3}{3}$
- None of these
- $\frac{\ln 3-4}{3}$