Section 4.6 Compound Interest Rate

(Based on 4.5)

Compound Interest Formula

- $A_n(t) = P(1+\frac{r}{n})^{nt}$

Where
P = Principal (Present Value)

$A_n(t)$ =Amount (Future Value with n compounds per year after t years)

r = nominal annual rate of interest (in DECIMAL form)
n = number of compounds per year
t = number of years

- Related Problems

Notice from the compound interest formula that there are 5 variables. Given any 4 of them, we can set up an equation for the 5th and solve for it by what we have learned from solving logarithmic and exponential equations.

Exercise 1
[4.6.1aPT]Find the amount that results from $5,000 invested at 7% compounded daily after a period of 4 years.

○ $\frac{5,000}{(1+\frac{0.07}{365})^{1460}}$

○ $\frac{5,000}{(1+\frac{0.07}{365})^{4}}$

○ $5,000(1+\frac{0.07}{365})^{4}$

○ $5,000(1+\frac{0.07}{365})^{1460}$

Exercise 2
[4.6.1bPT]If an investment pays 11% compounded semiannually, how much should you deposit now to have $1,700 in twelve years?

○ $1,700(1+\frac{0.11}{2})^{24}$

○ $1,700(1+\frac{0.11}{2})^{12}$

○ $\frac{1,700}{(1+\frac{0.11}{2})^{24}}$

○ $\frac{1,700}{(1+\frac{0.11}{2})^{12}}$

Exercise 3

[4.6.2cPT]How long will it take money to double if it is invested at 3% compounded annually?

○ $\frac{\ln \frac{1}{2}}{\ln(1+0.03)}$

○ $\frac{\ln 2}{0.03}$

○ $\frac{\ln 2}{\ln(1+0.03)}$

○ $\frac{\ln 2}{12\ln(1+\frac{0.03}{12})}$

Exercise 4

[4.6.2cPT]How long will it take money to triple if it is invested at 12% compounded semiannually?

○ $\frac{\ln 3}{2\ln(1+\frac{0.12}{2})}$

○ $\frac{\ln \frac{1}{3}}{2\ln(1+\frac{0.12}{2})}$

○ $\frac{\ln 3}{\ln(1+\frac{0.12}{2})}$

○ $\frac{\ln 3}{0.12}$

Exercise 5

[4.6.2ePT]What interest rate will take an initial investment of $5,000 to $11,000 in 9 years with annual compounding?

○ $(\frac{11}{5})^{\frac{1}{9}} - 1$

○ $(\frac{11}{5})^{\frac{1}{9}} + 1$

○ $(\frac{5}{11})^{\frac{1}{9}} - 1$

○ $1 - (\frac{11}{5})^{\frac{1}{9}}$

Continuous Compounding

In the compound interest formula, A increases as n increases. What happens to A as $n \to \infty$? Does the future value (pay-off) go to infinity? NO, the compound interest formula approaches a finite number as $n \to \infty$. This is explained in the continuous compounding formula

$A_n(t) = P(1+\frac{r}{n})^{nt} \to Pe^{rt}$ as $n \to \infty$.

Exercise 6

[4.6.1cPT]What is the amount in 5 years of $6,000 invested at 10.6% compounded continuously?

○ $\frac{6{,}000}{e^{0.53}}$

○ $6,000e^{5.3}$

○ $\frac{6{,}000}{e^{5.3}}$

○ $6,000e^{0.53}$

Exercise 7

[4.6.1dPT]A note will pay $4,765 at maturity in 5 years. How much is the note worth now, assuming continuous compounding at 9.8%?

○ $\frac{4{,}765}{e^{4.9}}$

○ $4,765e^{0.49}$

○ $\frac{4{,}765}{e^{0.49}}$

○ $4,765e^{4.9}$

Exercise 8

[4.6.2bPT]How many years will it take for an investment of $20,000 to grow to $50,000? Assume a rate of interest of 13% compounded continuously.

○ $\frac{\ln(\frac{5}{2})}{0.13}$

○ $\frac{\ln(\frac{2}{5})}{0.13}$

○ $\frac{\ln 5}{0.13 \ln 2}$

○ $\frac{\ln(\frac{5}{2})}{\ln(1+0.13)}$

Exercise 9

[4.6.2dPT]What interest rate, compounded continuously, will double an investment in 8 years?

○ $\frac{\ln 2}{8}$

○ $\frac{\ln\left(\frac{1}{2}\right)}{8}$

○ $8 \ln 2$

○ $\frac{8}{\ln 2}$

Effective Rate

We know from the previous formulas that the actual rate of return from compounded or continuously compounded is usually higher than the **NOMINAL** annual rate due to the effect of compounding. This is equivalent to as if we are using some higher ACTUAL/**EFFECTIVE** annual (simple) rate without compounding. In another word, the **effective rate** of interest is the equivalent annual simple rate of interest that would yield the same amount as compounding after 1 year. The effective annual rate, r_E and the nominal annual rate, r_N , have the following relation.

$$P(1+\frac{r_N}{n})^{n\cdot 1} = P(1+\frac{r_E}{1})^{1\cdot 1} = P(1+r_E) \Rightarrow r_E = (1+\frac{r_N}{n})^n - 1$$

Exercise 10

[4.6.2aPT]Find the effective rate of interest for 15% compounded semi-annually.

○ $(1+0.15)^2$

○ $(1+0.15)^2 - 1$

○ $(1+\frac{0.15}{2})^2 - 1$

○ $(1+\frac{0.15}{2})^2$