Section 9.1 Conics

Conics and **conic sections** are the names ordinarily applied to the class of plots that include parabolas, ellipses, and hyperbolas.

Conics can be obtained in the following equivalent ways:

(1) The set of points obtained by intersecting a plane with a right circular cone.

(2) The set of all points $P = P(x, y)$ in the $xy-$plane satisfying the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ for some choice of constants A, B, C, D, E, and F.

(3) The set of all points $P = P(x, y)$ in the xy-plane such that $d(P, P_o) = kd(P, L_o)$ where $d(P, P_o)$ denotes the distance from P to the fixed point P_o, $d(P, L_o)$ denotes the distance from P to the fixed line L_o, and k is a fixed constant, and P_o does not lie on L_o.

Note: Just as the graph of the linear equation $Ax + By + C = 0$ is a line in the $xy-$plane, the graph of the quadratic equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is a conic in the $xy-$plane.

Degenerate Forms of Conics Degenerate forms can be obtained from the above, and degenerate forms can be graphs of no points, one point, a line, a pair of intersecting lines, or a circle.

Identifying Conics The conic given by the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ can be identified by the value of its **discriminant** $B^2 - 4AC$ as follows:

$B^2 - 4AC = 0$ Parabola or degenerate form

$B^2 - 4AC < 0$ Ellipse or degenerate form

$B^2 - 4AC > 0$ Hyperbola or degenerate form

The following illustrates how the various conics are obtained from the intersections of planes with a cone.