Section 9.3 Ellipse

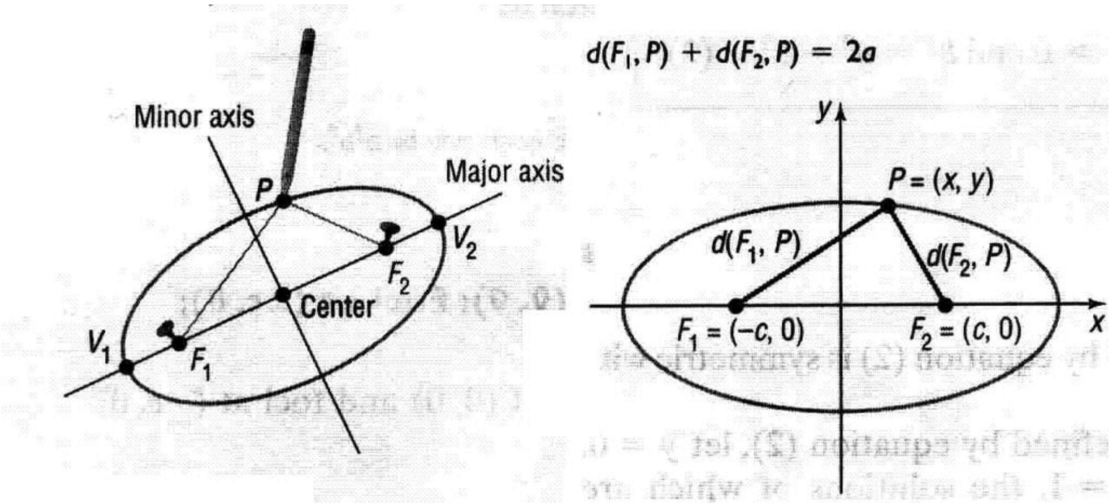
Application

Definition

Given two points F_1 and F_2 (**foci**) the distance of which is less than 2a, where a is a positive number. The set of all points P such that $d(P,F_1)+d(P,F_2)=2a$ is called an **ellipse**.

Note: d(A,B) denotes the distance between A and B. If $A=(x_A,y_A)$ and $B=(x_B,y_B)$ are two points, $d(A,B)=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}$.

Examples of ellipses:



The line containing the foci F_1 and F_2 is called the **major axis**.

The midpoint of the line segment joining the foci is called the **center** of the ellipse.

The line through the center and perpendicular (orthogonal) to the major axis is called the **minor axis**.

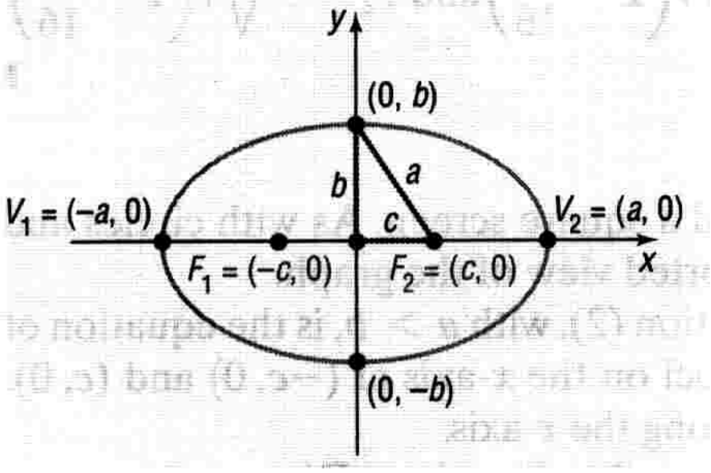
The 2 points there the major axis intersects with the ellipse are called **vertices**, denoted by V_1 and V_2.

Extra Credit

If $F_1=(-3,0)$, $F_2=(3,0)$ and a=5, use the definition of the ellipse and the formula of distance between two points to show that the equation of the ellipse is $\frac{x^2}{25}+\frac{y^2}{16}=1$.

Two Fundamental Forms of Ellipse (Pay attention to the differences and similarities.)

1. The **major axis is the x-axis (fat)** and the center is the origin.



From the graph we observe:

- a is HALF of the length of the MAJOR axis.
- The vertices are $V_1 = (-a, 0)$ and $V_2 = (a, 0)$.
- b is HALF of the length of the MINOR axis
- c is HALF of the distance between the foci or is the distance between the focus and the center.
- For an **ELLIPSE**, a, b and c have the following relation (Pythagorean Theorem):

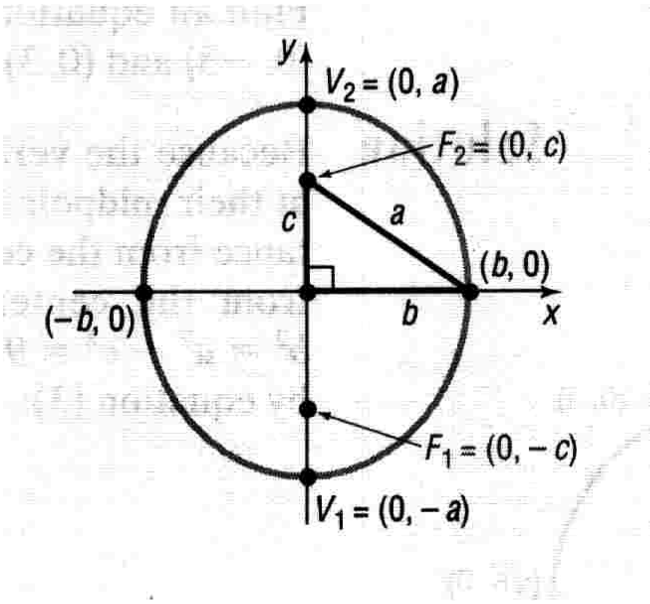
  $a^2 = b^2 + c^2$.

  NOTE: a>b.

Then the corresponding equation for the ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

2. The **major axis is the y-axis (thin)** and the center is the origin.



From the graph we observe: (almost the same as the fat one)

- a is HALF of the length of the MAJOR axis.
- The vertices are $V_1 = (0, -a)$ and $V_2 = (0, a)$.
- b is HALF of the length of the MINOR axis
- c is HALF of the distance between the foci or is the distance between the focus and the center.
- For an **ELLIPSE**, a, b and c have the following relation (Pythagorean Theorem):

  $a^2 = b^2 + c^2$.

  NOTE: a>b.

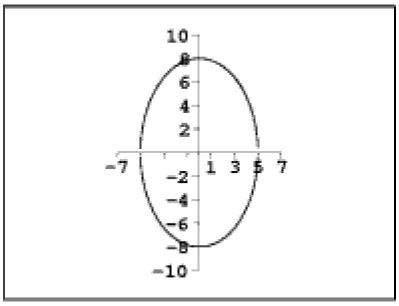
Then the corresponding equation for the ellipse is: (difference is here)

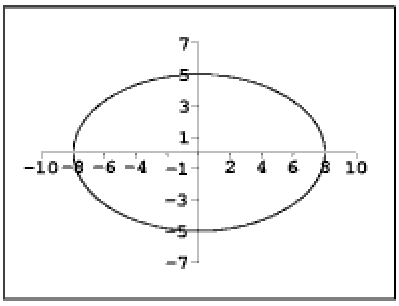
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

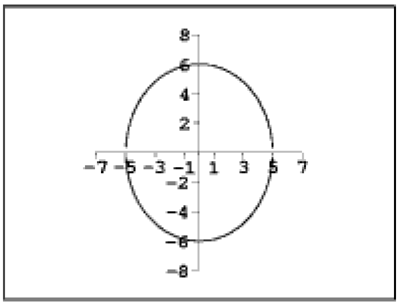
Think: But if we are given an equation like $\frac{x^2}{25} + \frac{y^2}{36} = 1$, how do we tell what is a and what is b?

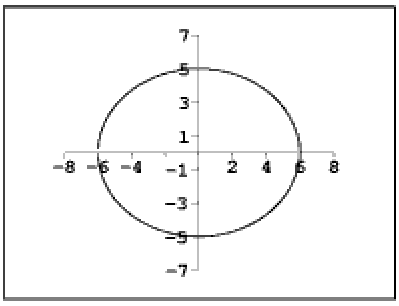
Example 1

[9.3.1aPT]Select the graph of $\frac{x^2}{36} + \frac{y^2}{25} = 1$.

○ 

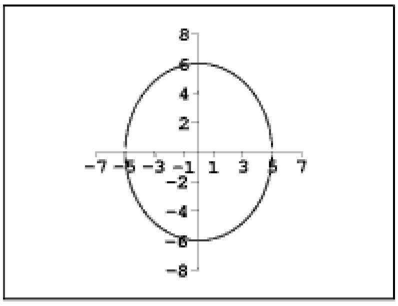
○ 

○ 

○ 

Exercise 2

[9.3.1bPT]Select the equation of the following graph.



○ $\frac{x^2}{25} + \frac{y^2}{36} = 1$

○ $\frac{x^2}{25} + \frac{y^2}{64} = 1$

○ $\frac{x^2}{64} + \frac{y^2}{25} = 1$

○ $\frac{x^2}{36} + \frac{y^2}{25} = 1$

Exercise 3

[9.3.2aPT]Select the equation of the ellipse with center at (0,0), focus at $(8, 0)$, and vertex at $(-10, 0)$.

○ $\frac{x^2}{100} + \frac{y^2}{64} = 1$

○ $\frac{x^2}{64} + \frac{y^2}{100} = 1$

○ $\frac{x^2}{100} + \frac{y^2}{36} = 1$

○ $\frac{x^2}{36} + \frac{y^2}{100} = 1$

Exercise 4

[9.3.2bPT]Find the foci of the ellipse given by $\frac{x^2}{27} + \frac{y^2}{36} = 1$.

○ $(0, \pm 3)$

○ $(\pm 3, 0)$

○ $(\pm 6, 0)$

○ None of these

○ $(0, \pm 6)$

Translated Forms

- If we replace x by x-h (h>0), then the graph of the equation is shifted right by h. If we replace x by x+h (h>0), then the graph of the equation is shifted left by h.
- Similarly, if we replace y by y-k (k>0), then the graph of the equation is shifted up by k. If we replace y by y+k (k>0), then the graph of the equation is shifted down by k.

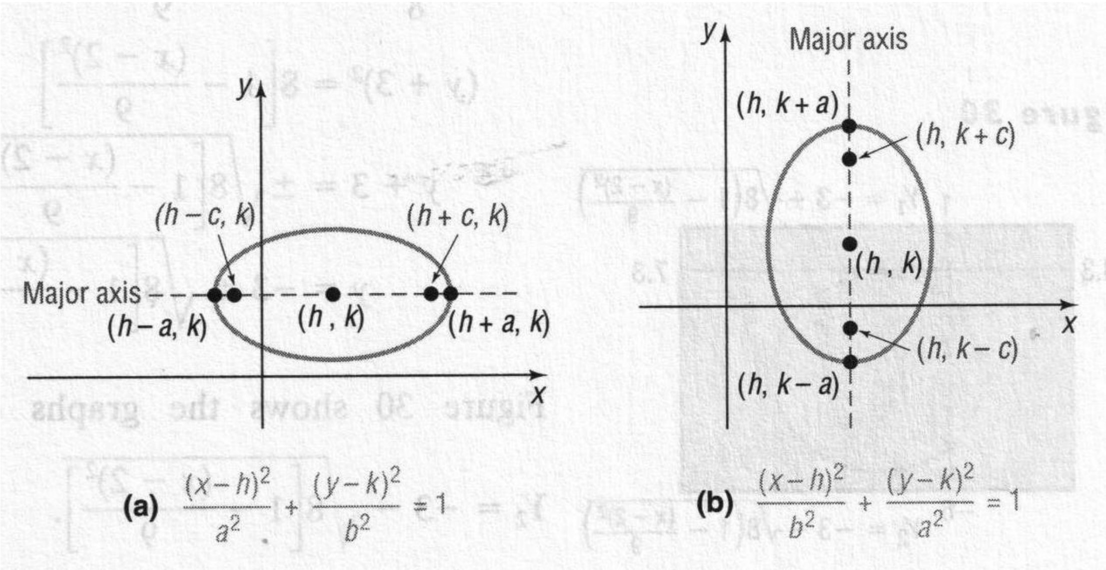
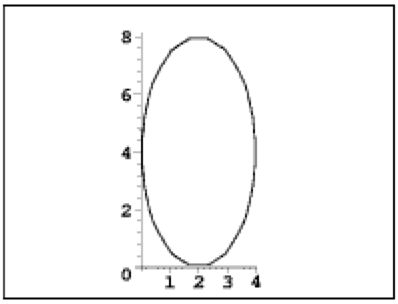


Table 3 Ellipses with Center at (h, k) and Major Axis Parallel to a Coordinate Axis

Center	Major Axis	Foci	Vertices	Equation
(h, k)	Parallel to x-axis	$(h + c, k)$	$(h + a, k)$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,$
		$(h - c, k)$	$(h - a, k)$	$a > b$ and $b^2 = a^2 - c^2$
(h, k)	Parallel to y-axis	$(h, k + c)$	$(h, k + a)$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1,$
		$(h, k - c)$	$(h, k - a)$	$a > b$ and $b^2 = a^2 - c^2$

Exercise 5

[9.3.1cPT]Select the equation of the following graph.



○ $\frac{(x-2)^2}{4} + \frac{(y-4)^2}{16} = 1$

○ $\frac{(x+4)^2}{16} + \frac{(y-2)^2}{4} = 1$

○ $\frac{(x-2)^2}{4} + \frac{(y+4)^2}{16} = 1$

○ $\frac{(x-4)^2}{16} + \frac{(y-2)^2}{4} = 1$

○ $\frac{(x+2)^2}{4} + \frac{(y+4)^2}{16} = 1$

○ $\frac{(x+2)^2}{4} + \frac{(y-4)^2}{16} = 1$

Exercise 6

[9.3.3aPT]Find the vertices of the ellipse given by $\frac{(x+5)^2}{27} + \frac{(y-2)^2}{36} = 1.$

○ $(-5 \pm 6, 2)$

○ $(-5, 2 \pm 3)$

○ $(-5, 2 \pm 6)$

○ $(-5 \pm 3, 2)$

Exercise 7

[9.3.3bPT]Find the foci of the ellipse given by $\frac{(x+5)^2}{36} + \frac{(y-2)^2}{27} = 1$.

○ $(-5, 2 \pm 6)$

○ $(-5 \pm 3, 2)$

○ $(-5 \pm 6, 2)$

○ $(-5, 2 \pm 3)$