Section 9.3 Hyperbola

(Pay Close Attention to the Difference from Ellipse)

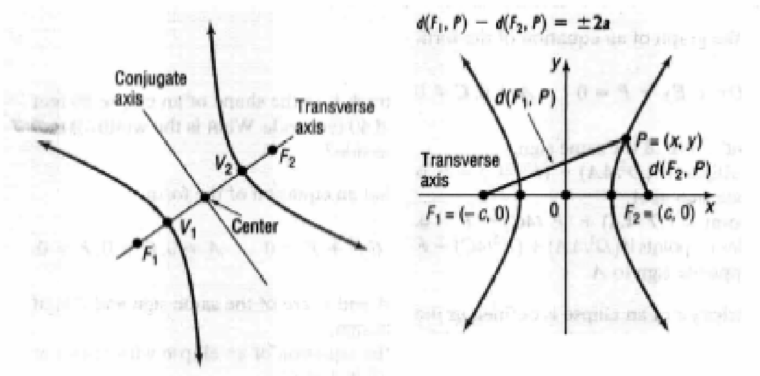
[Application]

Definition

Given any POSITIVE constant $2a$ and two fixed points F_1, F_2 (**foci**), the set of all points P such that $|d(P, F_1) - d(P, F_2)| = 2a$ is called a **hyperbola**.

Note: $d(A, B)$ denotes the distance from A to B: $d(A, B) = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$.

$|d(P, F_1) - d(P, F_2)| = 2a \Leftrightarrow d(P, F_1) - d(P, F_2) = \pm 2a$



The line containing the foci F_1, F_2 is called the **transverse axis**.

The midpoint of the line segment joining the foci is called the **center** of the hyperbola.

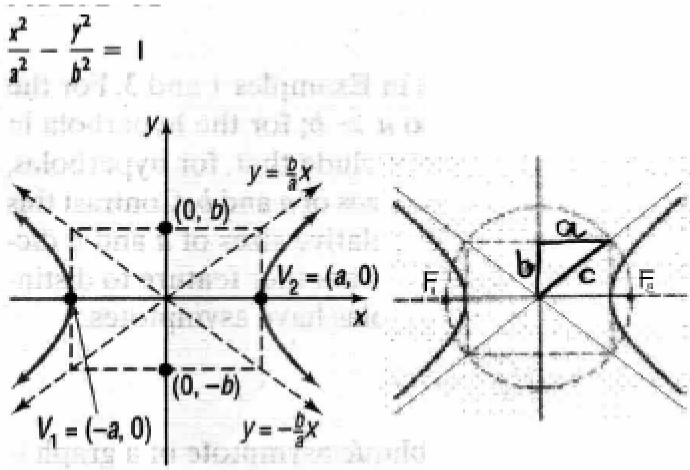
The line through the center and perpendicular to the transverse axis is called the **conjugate axis**.

The hyperbola consists of two separate curves, called **branches**.

The 2 points where the hyperbola intersects the transverse axis are the vertices, V_1, V_2 of the hyperbola.

Two Fundamental Forms of Hyperbolas (Pay attention to the differences and similarities.)

1. The **transverse axis is the x-axis** and the center is the origin.



An equation of the hyperbola whose

- transverse axis is the x-axis.
- The center is at the origin.
- The foci are $F_1 = (-c, 0)$ and $F_2 = (c, 0)$.
- The vertices are $V_1 = (-a, 0)$ and $V_2 = (a, 0)$.
- There are two OBLIQUE asymptotes $y = \pm\frac{b}{a}x$.

is:

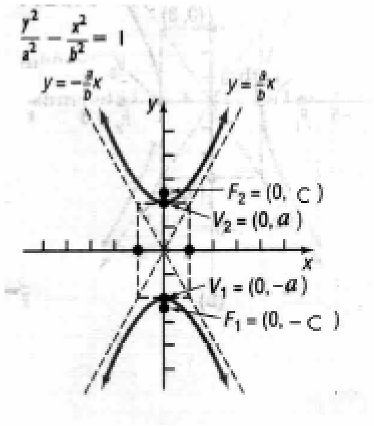
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where $a < c$.

- a is HALF of the distance between the vertices.
- b determines the slope of the hyperbola's asymptotes.
- c is HALF of the distance between the foci.

Then, $c^2 = b^2 + a^2$.

2. The **transverse axis is the y-axis** and the center is the origin.



An equation of the hyperbola whose

- transverse axis is the y-axis.
- The center is at the origin.
- The foci are $F_1 = (0, -c)$ and $F_2 = (0, c)$.
- The vertices are $V_1 = (0, -a)$ and $V_2 = (0, a)$.
- There are two OBLIQUE asymptotes $y = \pm\frac{a}{b}x$.

is:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

where $a < c$.

- a is HALF of the distance between the vertices.
- b determines the slope of the hyperbola's asymptotes.
- c is HALF of the distance between the foci.

Then, $c^2 = b^2 + a^2$.

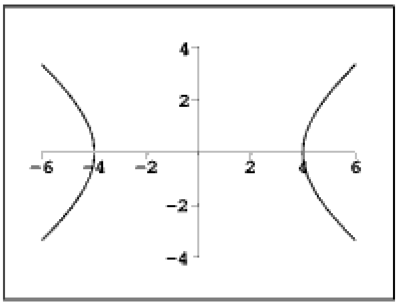
ATTN: a, b and c for the hyperbola have different relation as a, b and c for the ellipse.

Extra Credit:

How can you tell a from b, given the equation of an ellipse? How can you tell a from b, given the equation of a hyperbola? Do a, b and c denote the same things for the ellipse and the hyperbola? Which parameters have the same meaning? Which parameters have different meanings? (Hint: a and c denote the same thing for both the hyperbola and the ellipse. But b may have different meanings.)

Example 1

[9.4.1aPT]Select the graph of $\frac{x^2}{9} - \frac{y^2}{16} = 1$.



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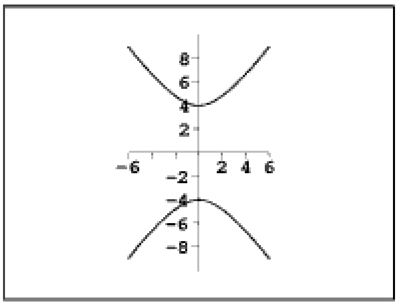

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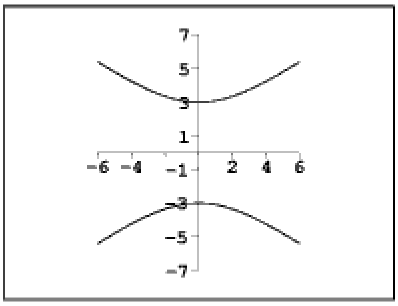

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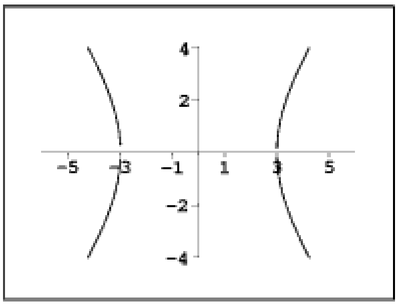

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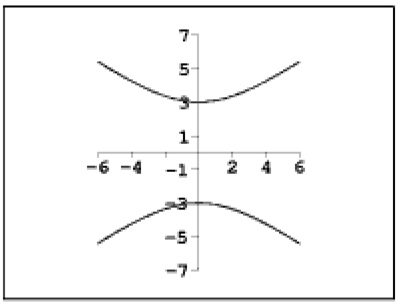
Exercise 2

[9.4.1bPT]Select the equation of the following graph.



○ $\frac{y^2}{9} - \frac{x^2}{16} = 1$

○ $\frac{x^2}{9} - \frac{y^2}{16} = 1$

○ $\frac{x^2}{16} - \frac{y^2}{9} = 1$

○ $\frac{y^2}{16} - \frac{x^2}{9} = 1$

Exercise 3

[9.4.2aPT]Select the equation of the hyperbola with center at (0,0), focus at $(-8,0)$, and vertex at $(-5,0)$.

○ $\frac{x^2}{25} - \frac{y^2}{39} = 1$

○ $\frac{x^2}{39} - \frac{y^2}{25} = 1$

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○ $\frac{y^2}{25} - \frac{x^2}{39} = 1$

Exercise 4

[9.4.2bPT]Select the foci of the hyperbola given by $\frac{y^2}{16} - \frac{x^2}{33} = 1$.

○ $(\pm 7, 0)$

○ $(\pm 4, 0)$

○ $(0, \pm 7)$

○ $(0, \pm 4)$

○ None of these

Exercise 5

[9.4.2cPT]Select the asymptotes of the hyperbola given by $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

○ $y = \pm \frac{4}{9} x$

○ $y = \pm \frac{4}{3} x$

○ $y = \pm \frac{9}{4} x$

○ $y = \pm \frac{3}{4} x$

Translated Forms

- If we replace x by x-h (h>0), then the graph of the equation is shifted right by h. If we replace x by x+h (h>0), then the graph of the equation is shifted left by h.
- Similarly, if we replace y by y-k (k>0), then the graph of the equation is shifted up by k. If we replace y by y+k (k>0), then the graph of the equation is shifted down by k.

Exercise 6

[9.4.3aPT]Select the vertices of the hyperbola given by $\frac{(y-5)^2}{16}-\frac{(x+1)^2}{33}=1$.

○ $(-1, 5 \pm 7)$

○ $(-1, 5 \pm 4)$

○ $(-1 \pm 7, 5)$

○ $(-1 \pm 4, 5)$

Exercise 7

[9.4.3bPT]Select the foci of the hyperbola given by $\frac{(x+3)^2}{25}-\frac{(y-4)^2}{39}=1$.

○ $(-3, 4 \pm 5)$

○ $(-3 \pm 8, 4)$

○ $(-3 \pm 5, 4)$

○ $(-3, 4 \pm 8)$