

MAC1140 SEC29 HW 10-26-2007 10.3

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Due: 10-29-2007

Full Name: Fei Hua Sec#: \_\_\_\_\_ Extra Credit Attempted? \_\_\_\_\_

1. [10.3.1aPT] Choose the echelon form of the following matrix

$\begin{bmatrix} -3 & 1 & -2 \\ 2 & -4 & -4 \end{bmatrix}$   $\begin{bmatrix} \textcircled{1} & -\frac{1}{3} & \frac{2}{3} \\ 2 & -4 & -4 \end{bmatrix} R_1 \cdot (-\frac{1}{3})$   
  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$   $\begin{bmatrix} \textcircled{1} & -\frac{1}{3} & \frac{2}{3} \\ 0 & -\frac{10}{3} & -\frac{16}{3} \end{bmatrix} R_2 - 2 \cdot R_1$   $\begin{matrix} R_2 & 2 & -4 & -4 \\ -2R_1 & -2 & +\frac{2}{3} & -\frac{4}{3} \\ \hline & 0 & -\frac{10}{3} & -\frac{16}{3} \end{matrix}$   
  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$   $\begin{bmatrix} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & \textcircled{1} & \frac{8}{5} \end{bmatrix} R_2 \cdot (-\frac{3}{10})$   
  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$   $\begin{bmatrix} 1 & 0 & \frac{6}{5} \\ 0 & 1 & \frac{8}{5} \end{bmatrix} R_1 + \frac{1}{3} \cdot R_2$   $\begin{matrix} R_1 & 1 & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3}R_2 & 0 & \frac{1}{3} & \frac{8}{5} \\ \hline & 1 & 0 & \end{matrix}$   
  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

*Already echelon form, but not among choices, because echelon form is not unique. We may try some other ERO while maintaining the structure of echelon form.*

2. [10.3.1bPT] Choose the reduced row echelon form of the following matrix

$\begin{bmatrix} 2 & -2 & -4 & -2 \\ 3 & -3 & -6 & -3 \\ -2 & 3 & 1 & 7 \end{bmatrix}$   $\begin{bmatrix} \textcircled{1} & -1 & -2 & -1 \\ 1 & -1 & -2 & -1 \\ -2 & 3 & 1 & 7 \end{bmatrix} \begin{matrix} R_1 \cdot \frac{1}{2} \\ R_2 \cdot \frac{1}{3} \end{matrix}$   
  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$   $\begin{bmatrix} \textcircled{1} & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -3 & 5 \end{bmatrix} R_3 + 2R_1$   $\begin{matrix} R_3 & -2 & 3 & 1 & 7 \\ 2R_1 & 2 & -2 & -4 & -2 \\ \hline & 0 & 1 & -3 & 5 \end{matrix}$   
  $\begin{bmatrix} 1 & 0 & 1 & -\frac{4}{5} \\ 0 & 1 & 0 & \frac{13}{5} \\ 0 & 0 & 0 & 0 \end{bmatrix}$   $\begin{bmatrix} \textcircled{1} & -1 & -2 & -1 \\ 0 & \textcircled{1} & -3 & 5 \\ 0 & \textcircled{1} & -3 & 5 \end{bmatrix} \begin{matrix} R_3 \\ R_2 \end{matrix}$   
  $\begin{bmatrix} 1 & 0 & 0 & -\frac{4}{5} \\ 0 & 1 & 0 & \frac{13}{5} \\ 0 & 0 & 0 & 0 \end{bmatrix}$   $\begin{bmatrix} 1 & 0 & -5 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_1 + R_2$   $\begin{matrix} R_1 & 1 & -1 & -2 & -1 \\ R_2 & 0 & 1 & -3 & 5 \\ \hline & 1 & 0 & -5 & 4 \end{matrix}$

$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 1 & 1 & -\frac{1}{5} \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -5 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3.

[10.3.1bPT] Choose the reduced row echelon form of the following matrix

$\begin{bmatrix} 2 & -4 & 1 & -4 \\ 4 & -8 & 7 & 2 \\ -2 & 4 & -3 & 5 \end{bmatrix}$

Handwritten work for the first matrix:

$$\begin{bmatrix} 2 & -4 & 1 & -4 \\ 4 & -8 & 7 & 2 \\ 0 & 0 & -2 & 9 \end{bmatrix} \begin{array}{l} R_2 - 2R_1 \\ R_3 + R_1 \end{array} \rightarrow \begin{bmatrix} 2 & -4 & 1 & -4 \\ 0 & 0 & 5 & 10 \\ 0 & 0 & -2 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -4 & 1 & -4 \\ 0 & 0 & 5 & 10 \\ 0 & 0 & -2 & 9 \end{bmatrix} \begin{array}{l} R_2 \cdot \frac{1}{5} \\ R_3 + R_2 \end{array} \rightarrow \begin{bmatrix} 2 & -4 & 1 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -4 & 1 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 11 \end{bmatrix} \begin{array}{l} R_1 - R_2 \\ R_3 - 3R_2 \end{array} \rightarrow \begin{bmatrix} 2 & -4 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -4 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 5 \end{bmatrix} \begin{array}{l} R_1 \cdot \frac{1}{2} \\ R_3 \cdot \frac{1}{5} \end{array} \rightarrow \begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} R_1 + 3R_3 \\ R_2 - 2R_3 \end{array} \rightarrow \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} R_1 + 2R_2 \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Handwritten work for the second matrix:

$$\begin{bmatrix} 1 & -2 & \frac{1}{2} & -2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & -2 & 1 \end{bmatrix} \begin{array}{l} R_2 \cdot \frac{1}{2} \\ R_3 + R_2 \end{array} \rightarrow \begin{bmatrix} 1 & -2 & \frac{1}{2} & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & \frac{1}{2} & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{array}{l} R_1 - \frac{1}{2}R_2 \\ R_3 \cdot \frac{1}{3} \end{array} \rightarrow \begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} R_1 + 3R_3 \\ R_2 - R_3 \end{array} \rightarrow \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} R_1 + 2R_2 \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.

[10.3.3aPT] Select the matrix which is in reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 1, 2, 4,$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad 3, 2, 1$$

○ None of these

$$\begin{bmatrix} 1 & 0 & 5 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

5.

[10.3.4aPT] Choose the correct  $x$ ,  $y$ , or  $z$  value for the solution of the system

$$\begin{cases} 3x + 2y - z = -7 \\ 2x - y - z = 3 \\ x + 3y - 2z = -6 \end{cases}$$

$z = -2$

$y = 2$

$y = 1$

$z = -1$

$x = -4$

$$\begin{cases} x + 3y - 2z = -6 & R_3 \\ 2x - y - z = 3 & R_2 \\ 3x + 2y - z = -7 & R_1 \end{cases}$$

$$\begin{cases} x + 3y - 2z = -6 \\ -7y + 3z = 15 & R_2 - 2R_1 \\ -7y + 5z = 11 & R_3 - 3R_1 \end{cases}$$

$$\begin{array}{r} R_2 \quad 2x - y - z = 3 \\ -2R_1 \quad -2x - 6y + 4z = -12 \\ \hline -7y + 3z = 15 \end{array}$$

$$\begin{array}{r} R_3 \quad 3x + 2y - z = -7 \\ -R_1 \quad -3x - 9y + 6z = 18 \\ \hline -7y + 5z = 11 \end{array}$$

$$\begin{cases} x + 3y - 2z = -6 \\ -7y + 3z = 15 \\ 2z = -4 & R_3 - R_2 \end{cases}$$

$$\Rightarrow \begin{cases} x = -1 \\ y = -3 \\ z = -2 \end{cases} \quad \text{unique soln.}$$