1 Slide Sets

- Material on vibrations is covered in Sets 13, 14, and 15 and textbook sections 3.7 and 3.8.
- Material on Laplace transforms is covered in Sets 16, 17, 18 and 19 and textbook Chapter 6.
- Material on systems of first order ODEs is covered in Sets 20 and 21 and pieces of textbook sections 7.1, 7.4, 7.5 and 7.6. The associated material on linear algebra is in textbook sections 7.2, and 7.3.

2 Differential Equations for Mechanical Vibrations

- $mu'' + \gamma u' + ku = f(t)$ form
- differential equations from modeling of mass-spring system
  - $F = ma$
  - k spring constant determines force due to spring attempting to return to equilibrium
  - m mass of object combines with e.g. gravity to exert a force on system
  - γ damping of system due to e.g. atmospheric drag
- types of vibration
  - undamped free vibrations: $mu'' + ku = 0$
  - damped free vibrations $mu'' + \gamma u' + ku = 0$
  - undamped forced vibrations $mu'' + ku = f(t)$
  - damped forced vibrations $mu'' + \gamma u' + ku = f(t)$

3 Behavior of Mechanical Vibrations

3.1 Undamped Free Vibrations

- ODE $mu'' + ku = 0$
- persistent sinusoidal oscillation
- natural frequency ω_0
- natural period T

- amplitude/phase representation of solution: $u(t) = R\cos(\omega_0 t - \delta)$
- trends in solution behavior as parameters of system change

3.2 Damped Free Vibrations

- ODE $mu'' + \gamma u' + ku = 0$
- quasi-frequency μ as a function of system parameters
- quasi-period T_d as a function of system parameters
- amplitude/phase representation of solution
- type of damping
  - lightly or moderately damped
  - critical damping
  - overdamping
- relation of type of damping to the roots of the characteristic equation
- trends in decaying behavior of different types as a function of parameters of the system, e.g., quasi-frequency vs. natural frequency

3.3 Undamped Forced Vibrations

- ODE $mu'' + ku = F_0 \cos \omega t$
- various representations of solution
- relate forcing frequency to natural frequency
- "at rest" initial conditions and beat or amplitude modulation form of solution
- "at rest" initial conditions and $\omega \to \omega_0$ yielding resonance

3.4 Damped Forced Vibrations

- ODE $mu'' + \gamma u' + ku = F_0 \cos \omega t$
- general solution form $u(t) = u_h(t) + U(t)$
  - $u_h(t)$ is general homogeneous solution
  - $u_h(t)$ is a transient solution that decays
  - $U(t)$ is a particular solution

  - $u(t) \to U(t)$ and $U(t)$ called steady state solution or forced response.
- relationship between $F(t)$ and $U(t)$ and trends as a function of parameters of system
  - amplitude/phae form useful
  - low-frequency forcing
  - high-frequency forcing
  - forcing with $\omega \approx \omega_0$

4 Laplace Transform Methods

4.1 Basics

- definition of integral transforms
- improper integrals as limits of definite integrals
- definition of piecewise continuous functions
- definition of real form of Laplace transform
- basic function and their transforms

4.2 IVPs and Laplace Transforms

- linearity of Laplace transform
- relationship of a the Laplace transforms of a function and its derivative
- form of solution of a constant coefficient linear second order ODE

4.3 IVPs, Laplace Transforms, and Special Functions

- Heaviside function and its Laplace transform
- using the Heaviside function to specify pulses and other interesting functions
- impulse of a pulse function
- Dirac Delta function and its Laplace transform
- solving IVPs with forcing functions related to Heaviside and Dirac delta function

4.4 Laplace Transforms and System Theory

- convolution integral and the product of Laplace transforms
- definition of the transfer function of a linear second order ODE
- definition of the impulse function of a linear second order ODE
- solution of ODE interpretted in time and Laplace domains using convolution and the transfer function

5 Systems of First Order ODEs

- definition of a system of first order ODEs (nonlinear and linear)
- existence and uniqueness conditions for soltion of system of linear first order ODEs
- general solution conditions and form
- linear independence/dependence of vectors and vector functions on an interval
- eigenvalues and eigenvectors of a matrix $A \in \mathbb{R}^{n \times n}$
- computing eigenvalues and eigenvectors of a matrix $A \in \mathbb{R}^{2 \times 2}$
- homogeneous problem for a system of linear first order ODEs with constant coefficients
  - form of general solution with distinct and real eigenvalues for $A \in \mathbb{R}^{n \times n}$
  - computing the solution with distinct and real eigenvalues for $A \in \mathbb{R}^{2 \times 2}$
  - form of general solution with a complex conjugate pair of eigenvalues and distinct and real eigenvalues for $A \in \mathbb{R}^{n \times n}$
  - computing the solution with complex conjugate eigenvalues when for $A \in \mathbb{R}^{2 \times 2}$