ALGEBRA REVIEW

RECALL : EXPONENTIAL AND LOGARITHMIC LAWS/PROPERTIES

$* \; For\ a\ and\ b\ positive,\ a \neq 1, b \neq 1,\ and\ n, m, x, and\ y\ real$

$\bullet\, a^n a^m = a^{n+m}$ $\bullet\, (a^n)^m = a^{nm}$ $\bullet\, a^n b^n = (ab)^n$ $\bullet\, a^0 = 1$ $\bullet\, \frac{a^n}{a^m} = a^{n-m}$ $\bullet\, a^{-n} = \frac{1}{a^n} = (\frac{1}{a})^n$

$\bullet\, a^x = a^y\ \ if\ and\ only\ if\ \ x = y$ $\bullet\, a^x = b^x\ \ if\ and\ only\ if\ \ a = b\ ,\ x \neq 0$

$* \; For\ x\ and\ y\ are\ positive,\ a > 0,\ and\ a \neq 1,\ c = real$

$\bullet\, \log_a x = y\ \ if\ and\ only\ if\ \ x = a^y$ $\bullet\, \log_a x = \log_a y\ \ if\ and\ only\ if\ \ x = y$

$\bullet\, \log_a xy = \log_a x + \log_a y$ $\bullet\, \log_a \frac{x}{y} = \log_a x - \log_a y$ $\bullet\, \log_a x^c = c \log_a x$ $\bullet\, \log_y x = \frac{\log_a x}{\log_a y}\ ,\ y \neq 1$

$\bullet\, \log_a a = 1$ $\bullet\, \log_a 1 = 0$ $\bullet\, \log_a a^x = x$ $\bullet\, \log_a 0\ \ and\ \ \log_a (negative\ number)\ \ are\ undefined$

$\bullet\, a^{\log_a x} = x$ $\bullet\, a^{n \log_a x} = x^n\ \ , n\ is\ real$

NOTE: $* \log x$ *is called the common* $\log\ (\log_{10} x)$ $* \ln x$ *is called the natural* $\log\ (\log_e x)$

$\bullet\, \log 10 = 1$ $\bullet\, \ln e = 1$ $\bullet\, e^{\ln x} = x$ $\bullet\, \ln x = y\ \ if\ and\ only\ if\ \ x = e^y$ $\bullet\, \log_y x = \frac{\ln x}{\ln y} = \frac{\log x}{\log y},\ y \neq 1$

NOTE: * Assume all the radicals are well defined in the real number system.
* In this work sheet , we deal with the real number system only.

1-Solve for x:

A- $\frac{1}{2}x + 7 = 2x - 3$

B- $\frac{4}{3}(x+8) = \frac{3}{4}(2x+12)$

2- Solve the following inequalities:

A- $5 - 5x < -3x + 1$

B- $3(x-1) \geq 2(1-x)$

C- $(x^2+2)(4-x)(x+1) > 0$

D- $\frac{x-10}{3(x+2)(1-5x)} < 0$

3-Classify each statement as true or false. If false, correct the right side of the equality to obtain a true statement.

A- $3^4 \cdot 3^2 = 3^8$, **B-** $(2^3)^4 = 2^7$, **C-** $2^5 \cdot 2^2 = 4^7$, **D-** $\frac{10^4}{5^4} = 2^4$, **E-** $(2^0)^3 = 2^3$, **F-** $3^4 + 3^4 = 3^8$

G- $(a^2 b)^3 = a^2 b^3$, **H-** $(a+b)^0 = a+1\ ;\ a \neq 0$, **I-** $\sqrt{a} + \sqrt{a} = \sqrt{a^2}$, **J-** $\frac{1}{2^{-3}} = -2^3$, **K-** $(2+\pi)^{-2} = \frac{1}{4} + \frac{1}{\pi^2}$

L- $\frac{2^{-5}}{2^3} = 2^{-2}$, **M-** $\frac{a}{2\sqrt{a}} = 2\sqrt{a}$, **N-** $\frac{1}{-3x^{-2}} = 3x^2$, **O-** $(a^{\frac{1}{2}} + b^{\frac{1}{2}})^2 = a + b$

4- Evaluate the following:

A- -2^4 , **B-** $(\frac{1}{2})^3(-2)^3$, **C-** $\frac{(-2)^{-3}}{(-2)^{-5}}$, **D-** $\frac{2^{10}}{4^3}$, **E-** $(\frac{2}{3})^{-2} + (\frac{2}{3})^{-1}$, **F-** $[(-7)^2(-3)^2]^{-1}$

5-Simplify and express each answer using positive exponents only.

A- $(x^{-3})^2$, **B-** $(-2x^3y^{-1})^{-1}(-3x^2y^3)^2$, **C-** $(\frac{3a^2}{b^3})^{-2}(\frac{-2a}{3b})^3$, **D-** $-7(2x^2-3x)^{-5}(2x-3)$, **E-** $x^{-2} + y^{-2}$

F- $\dfrac{(49a^{-4})^{-\frac{1}{2}}}{(27b^{6})^{-\frac{1}{3}}}$, **G-** $\dfrac{1}{2}(x^2+4x)^{-\frac{1}{2}}(2x+4)$

6- Evaluate the following:

A- $(-64)^{\frac{1}{3}}$, **B-** $(-125)^{-\frac{2}{3}}$, **C-** $\dfrac{9^{\frac{1}{2}}}{\sqrt[3]{27}}$, **D-** $(\dfrac{1}{8}+\dfrac{1}{27})^{\frac{1}{3}}$, **E-** $(\dfrac{8}{27})^{-\frac{2}{3}}+(-\dfrac{32}{243})^{\frac{2}{5}}$, **F-** $(-\dfrac{16}{81})^{-\frac{3}{4}}+(\dfrac{36}{49})^{-\frac{1}{2}}$

7- Simplify the following:

A- $\sqrt{2}+\sqrt{18}$, **B-** $\sqrt{6}\cdot\sqrt{12}$, **C-** $\dfrac{8}{\sqrt{2}}+2\sqrt{50}$, **D-** $\sqrt[3]{-54x}+\sqrt[3]{250x}$

8- Simplify by performing the indicated operation.

A- $5y-[y-(3y+8)]$, **B-** $2x^2(2x+1-10x^2)$, **C-** $-4x(x^4-\dfrac{1}{4}x^3+4x^2-\dfrac{1}{16}x+1)$

D- $(3-2x)(3+2x)$, **E-** $(-2x-3)(3x+6)$, **F-** $(7-3x)(4x-9)$, **G-** $(\sqrt{x}-10)(\sqrt{x}+10)$

H- $(\sqrt{3}-\sqrt{x})(\sqrt{3}+\sqrt{x})$, **I-** $(2-x)(x^2+2x-3)$, **J-** $(2x-3y)^2$

9- Rationalize the denominator and simplify

A- $\dfrac{12}{\sqrt{5}-\sqrt{3}}$, **B-** $\dfrac{20}{3-\sqrt{2}}$, **C-** $\dfrac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}$, **D-** $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$, **E-** $\dfrac{x}{\sqrt{3}-\sqrt{x+3}}$, **F-** $\dfrac{9x-x^2}{\sqrt{x}+3}$

10- Rationalize the numerator and simplify

A- $\dfrac{\sqrt{5}+3}{\sqrt{5}}$, **B-** $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}$, **C-** $\dfrac{\sqrt{4+h}-2}{h}$, **D-** $\dfrac{\sqrt{x}-2}{x-4}$

11- Factor completely, and simplify if possible.

A- $8a^2-2b^2$, **B-** $4-\dfrac{x^2}{y^4}$, **C-** a^8-b^8 , **D-** $4-y^2+4x-xy^2$, **E-** $4x^2-9$,**F-** $24x^3-18x^2$

G- $3x^4-6x^3+12x$,**H-** $(1+x)^2(-1)+(1-x)(2)(1+x)$, **I-** $(x^2+2)^2(3)+(3x-1)(x^2+2)(2x)$

J- $(2-x)^3(2)(2x+1)(2)+(2x+1)^2(3)(2-x)^2(-1)$

12- Factor each trinomial

A- $6x^2-7x-3$, **B-** $6x^2-19x+10$, **C-** $6x^2+13x-5$, **D-** $15x^2-14x-8$,**E-** $6x^2-5x-21$

F- $9x^2-18x+5$, **G-** $15x^2+19x-56$

13- Take each expression at the left and change it into the equivalent form at the right.

A- $x^{\frac{1}{2}}+(x-4)\dfrac{1}{2}x^{-\frac{1}{2}}$; $\dfrac{3x-4}{2\sqrt{x}}$

B- $x^{\frac{1}{3}}+(x-1)\dfrac{1}{3}x^{-\frac{2}{3}}$; $\dfrac{4x-1}{3\sqrt[3]{x^2}}$

C- $(x+1)^{\frac{1}{2}}(2)+(2x+1)\dfrac{1}{2}(x+1)^{-\frac{1}{2}}$; $\dfrac{6x+5}{2\sqrt{x+1}}$

D- $\frac{1}{2}(1-x)^{-\frac{1}{2}}(-2x)+(1-x)^{\frac{1}{2}}$ **;** $\frac{1-2x}{\sqrt{1-x}}$

14- Perform the indicated operations and simplify.

A- $\frac{x^2}{x^2+2x+1}+\frac{x-1}{3x+3}-\frac{1}{6}$, **B-** $\frac{1+\frac{3}{x}}{x-\frac{9}{x}}$, **C-** $\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$, **D-** $\frac{1-\frac{y^2}{x^2}}{1-\frac{y}{x}}$, **E-** $\frac{\frac{1}{x+3}-\frac{1}{3}}{x}$

F- $\frac{\frac{1}{4}-\frac{1}{x^2}}{x-2}$ **,** **G-** $\frac{x^{-2}-\frac{1}{4}}{\frac{1}{x}-\frac{1}{2}}$ **, H-** $\frac{\frac{1}{4+h}-\frac{1}{4}}{h}$, **I-** $\frac{\frac{5}{x^2-4}}{\frac{10}{x-2}}$ **,** **J-** $\frac{3}{x^2+x}+\frac{2}{x^2-1}$

K- $\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}$ **,** **L-** $\frac{(x^2-9)(2x)-x^2(2x)}{(x^2-9)^2}$ **,** **M-** $\frac{x^2(4-2x)-(4x-x^2)(2x)}{x^4}$

N- $\frac{(x+1)^2(2x)-(x^2-1)(2)(x+1)}{(x+1)^4}$ **, O-** $\frac{x}{(1-x)^2-1}$ **,** **P-** $\frac{(x-2)^2-4}{2x}$ **, Q-** $\frac{x-2}{(x-3)^2-1}$

15- Classify each statement as true or false. If false, correct the right side of the equality to obtain a true statement.

A- $\frac{5}{7}-\frac{2}{3}=\frac{3}{4}$, **B-** $\frac{2x+y}{y-2x}=-2(\frac{x+y}{x-y})$ **,** **C-** $\frac{3ax-5b}{6}=\frac{ax-5b}{2}$ **,** **D-** $\frac{x+x^{-1}}{xy}=\frac{x+1}{x^2y}$

E- $x^{-1}+y^{-1}=\frac{y+x}{xy}$ **,** **F-** $\frac{2}{\frac{3}{4}}=\frac{8}{3}$ **,** **G-** $\frac{\frac{3}{2}}{5}=\frac{15}{2}$ **,** **H-** $\frac{\frac{3}{4}}{\frac{2}{5}}=\frac{15}{8}$**,** **I-** $(\frac{3}{2+a})^{-1}=\frac{2}{3}+\frac{a}{3}$ **,**

J- $2\sqrt{x+1}=\sqrt{2x+2}$ **,** **K-** $If \quad 4x+y=84 \quad then \quad y=\frac{84}{4x}$

16- Write each expression in the form $ax^p+bx^q+cx^r+\cdots$ where $a,b,c,p,q,and\ r$ are real numbers.

EXAMPLE:

I- $\frac{2x^5-3x^3-x^2+5}{2x^4}=\frac{2x^5}{2x^4}-\frac{3x^3}{2x^4}-\frac{x^2}{2x^4}+\frac{5}{2x^4}=x-\frac{3}{2}x^{-1}-\frac{1}{2}x^{-2}+\frac{5}{2}x^{-4}$

II- Another method:

$$\frac{2x^5-3x^3-x^2+5}{2x^4}=(2x^5-3x^3-x^2+5)\frac{1}{2}x^{-4}=\frac{2}{2}x^5x^{-4}-\frac{3}{2}x^3x^{-4}-\frac{1}{2}x^2x^{-4}+\frac{5}{2}x^{-4}=x-\frac{3}{2}x^{-1}-\frac{1}{2}x^{-2}+\frac{5}{2}x^{-4}$$

A- $\frac{5x^6-2x^4}{3x^7}$ **, B-** $\frac{3x-2x^4+5x^5}{x^3}$**,** **C-** $\frac{3x^2-2\sqrt{x}-x+3}{2\sqrt{x}}$ **, D-** $\frac{1-\sqrt{x}+2x^2-3x}{\sqrt[3]{x}}$ **, E-** $\frac{(x^2+2)^2}{x^5}$

17- Solve for x:

A- $(2x-1)^2=4$ **, B-** $3x^2-21=0$ **, C-** $6x^2-5x-21=0$**, D-** $x^2=2x$ **,** **E-** $9x^2-6=15x$

F- $\sqrt{x+1}=3$**,** **G-** $\sqrt{x+2}=\sqrt{2x-5}$**,** **H-** $\frac{4}{\sqrt{x-1}}=2$**,** **I-** $2x^2-2x-1=0$**,** **J-** $x^2-2x-2=0$

18- Find the domain of $f(x)$

A- $f(x)=\frac{x}{x^2+1}$ **,** **B-** $f(x)=\frac{2x}{x^2-4}$ **,** **C-** $f(x)=\sqrt{3-2x}$ **,** **D-** $f(x)=\frac{x}{\sqrt{2x+4}}$ **, E-** $f(x)=\sqrt[3]{3x-5}$

E- $f(x)=\dfrac{3}{\sqrt[5]{2-6x}}$ **, G-** $f(x)=3x+7$ **, H-** $f(x)=x^3-2x+1$

19- Solve the following for x:

A- $\dfrac{(x+2)^3(2)(x-1)-(x-1)^2(3)(x+2)^2(1)}{(x+2)^6}=0$

B- $\dfrac{(x^2-2)^2(2)(1-2x)(-2)-(1-2x)^2(2)(x^2-2)(2x)}{(x^2-2)^4}=0$

C- $(2-3x)^2(2)(2x-4)(2)+(2x-4)^2(2)(2-3x)(-3)=0$

D- $(1-2x)(3)(x-4)^2(1)+(x-4)^3(-2)=0$

E- $8-\dfrac{2}{x^2}=0$, **F-** $-\dfrac{1}{x^2}-\dfrac{2}{x^3}=0$ **, G-** $\dfrac{1}{2}x^{-\frac{1}{2}}-1=0$, **H-** $\dfrac{1}{2}x(1-x)^{-\frac{1}{2}}+3\sqrt{1-x}=0$

20- If $f(x)=\dfrac{-x^2-2}{x-\sqrt{2}}$, find $f(-\sqrt{2})$

21- If $f(x)=\dfrac{3-x^2}{\sqrt{3}-x}$, find $f(-\sqrt{3})$

22- If $f(x)=\dfrac{\sqrt{x}-2}{2x-4}$, find $f(4)$

23- If $f(x)=\dfrac{1}{2}x^{-\frac{1}{2}}-x^{-2}$, find $f(\dfrac{1}{4})$

24- If $f(x)=\dfrac{x}{2}-\dfrac{2}{x}$, find **A-** $f(\dfrac{1}{2})$, **B-** $f(-1)$ **, C-** $f(-\dfrac{2}{3})$

25- If $f(x)=1-2x-3x^2$, find $f(-1)$

26- If $f(x)=2-3x-x^2$, **a)** Find $f(-2)$, **b)** Find $f(-2+h)$, **c)** Evaluate $\dfrac{f(-2+h)-f(-2)}{h}$

27- If $f(x)=x-2x^2$, **a)** Find $f(1)$, **b)** Find $f(1+h)$, **c)** Evaluate $\dfrac{f(1+h)-f(1)}{h}$

28- If $f(x)=2-\dfrac{1}{x}$, **a)** Find $f(3)$, **b)** Find $f(3+h)$, **c)** Evaluate $\dfrac{f(3+h)-f(3)}{h}$

29- If $f(x)=\dfrac{2}{x-1}$, **a)** Find $f(-1)$, **b)** Find $f(-1+h)$, **c)** Evaluate $\dfrac{f(-1+h)-f(-1)}{h}$

30- For the following functions:

A- $f(x)=2-x-x^2$, **B-** $f(x)=2x-3$, **C-** $f(x)=2x^2-x$, **D-** $f(x)=\dfrac{x+1}{3}$, **E-** $f(x)=-\dfrac{4}{x}$

I) Find $f(x+h)$, **II)** Evaluate $\dfrac{f(x+h)-f(x)}{h}$

31- Classify each statement as true or false. If false, correct the right side of the equality to obtain a true statement.

A- $\log_2 2^{-1}$ *is undefined* **B-** $\log x\cdot\log y=\log xy$ **C-** $\log 3^2=(\log 3)^2$ **D-** $\log_2 3=2^3$ **E-** $\log(-\dfrac{1}{2})=\log 2$

F- $\log_{\frac{1}{2}}1=0$ **, G-** $\ln\dfrac{1}{e}=-1$ **, H-** $e^{\ln\frac{1}{3}}=3$ **, I-** $e^{-\ln 3}=-3$ **, J-** $e^{\ln(-2)}=-2$ **, K-** $\log(x+y)=\log x+\log y$

L- $\dfrac{\log x}{\log y}=\log\dfrac{x}{y}$,**M-** $\log x-\log y=\log(x-y)$, **N-** $\log_3 3^{-2}=\dfrac{1}{9}$, **O-** $\log 0=1$, **P-** $If\ \log_a 2=x\ \ then\ \ x=2^a$

Q- $(x)(\ln x)=\ln x^2$, **R-** $(5-2\ln x)(x)=5x-2\ln x^2$

32- Solve for x

A- $\dfrac{1}{e^{x-1}}=e^2$, **B-** $(0.01)^x=1000$, **C-** $7^{x^2+x}=49$, **D-** $(\dfrac{9}{25})^x=\dfrac{5}{3}$, **E-** $\log_x\dfrac{27}{8}=-3$, **F-** $e^{\ln(x^2-x)}=6$

G- $\log_{16}x+\log_{16}(x-4)=\dfrac{5}{4}$, **H-** $\log(3-x)-\log(12-x)=-1$, **I-** $\ln x=\dfrac{1}{2}\ln 4+\dfrac{2}{3}\ln 8$, **K-** $4^{2x}=5$

L- $e^{1-x}=2$, **M-** $2^{x-2}=3$, **N-** $\ln\dfrac{1}{e^3}=2x$, **O-** $\log_x(2x+15)=2$

33- If $f(x)=e^{2x}-e^{-x}$, find $f(\ln 2)$

34- If $f(x)=2^x$, find **A-** $f(-1)$, **B-** $f(\dfrac{1}{2})$, **C-** $f(-\dfrac{1}{2})$, **D-** $f(-\dfrac{2}{3})$

35- If $f(x)=4x-x\ln x$, find $f(e^3)$

36- If $f(x)=\dfrac{x^2-1}{e^x}$, find **A-** $f(-1)$, **B-** $f(2)$

37- If $f(x)=\dfrac{1-2\ln x}{x}$, find **A-** $f(e)$, **B-** $f(e^{\frac{2}{3}})$, **C-** $f(-e)$

38- Solve for x

A- $4-(x\cdot\dfrac{1}{x}+\ln x)=0$, **B-** $3-[2x\cdot\dfrac{1}{x}+(\ln x)(2)]=0$, **C-** $\dfrac{e^x\cdot 2x-x^2\cdot e^x}{e^{2x}}=0$, **D-** $\dfrac{x\ \cdot e^{-x}(-1)-e^{-x}(2x)}{x^4}=0$

G- $\dfrac{x\cdot\dfrac{2}{x}-(1+\ln x)}{x^2}=0$, **F-** $\dfrac{x^2(-\dfrac{2}{x})-(1-2\ln x)(2x)}{x^4}=0$, **G-** $\dfrac{(x^2-8)e^x-e^x(2x)}{(x^2-8)^2}=0$

H- $(1-x)e^{2x}(2)+e^{2x}(-1)=0$, **I-** $x^2\cdot\dfrac{1}{x}+(\ln x)(2x)=0$, **J-** $(3-x^2)e^{-x}(-1)+e^{-x}(-2x)=0$

K- $\dfrac{(\ln x)(2x)-x^2\cdot\dfrac{1}{x}}{(\ln x)^2}=0$, **L-** $x^2\cdot\dfrac{2}{x}-(2\ln x-1)(2x)=0$, **M-** $(x-2)e^{-2x}(-2)+e^{-2x}=0$

39- Evaluate the following . (**Do not use decimals)**

A- $f(2)-f(-3)$, given $f(x)=6x-\dfrac{x^3}{3}-\dfrac{x^2}{2}$

B- $f(1)-f(-2)$, given $f(x)=4x-x^2-\dfrac{2}{3}x^3$

C- $f(2)-f(-1)$, given $f(x)=\dfrac{1}{3}x^3+\dfrac{1}{4}x^2+2x$

D- $f(0)-f(-2)$, given $f(x)=\dfrac{1}{3}x^3-\dfrac{3}{2}x^2$

E- $f(3)-f(-1)$, given $f(x)=3x+x^2-\dfrac{x^3}{3}$