

Sections 6.2 and 6.3

I) Complete each equation to get a specific case of an addition or subtraction or double angle or half angle formula.

1) $\sin 5^\circ \cos 12^\circ + \cos 5^\circ \sin 12^\circ = ?$ 2) $\cos \frac{3\pi}{10} \cos \frac{\pi}{5} + \sin \frac{3\pi}{10} \sin \frac{\pi}{5} = ?$ 3) $\frac{\tan(-15^\circ) - \tan(-20^\circ)}{1 + \tan(-15^\circ) \tan(-20^\circ)} = ?$

4) $\sin(\theta + 5^\circ) \cos(\theta - 5^\circ) + \cos(\theta + 5^\circ) \sin(\theta - 5^\circ) = ?$ 5) $\cos\left(\frac{x}{2} - 1\right) \cos\left(\frac{x}{2} + 1\right) - \sin\left(\frac{x}{2} - 1\right) \sin\left(\frac{x}{2} + 1\right) = ?$

6) $2 \sin 10^\circ \cos 10^\circ = ?$ 7) $1 - 2 \sin^2 5\theta = ?$ 8) $\cos^2 3\theta - \sin^2 3\theta = ?$ 9) $2 \cos^2 \frac{\theta}{2} - 1 = ?$

10) $\sqrt{\frac{1 - \cos 20^\circ}{2}} = ?$ 11) $\frac{2 \tan \frac{\pi}{14}}{1 - \tan^2 \frac{\pi}{14}} = ?$ 12) $\frac{1 - \cos \frac{\pi}{5}}{\sin \frac{\pi}{5}} = ?$ 13) $-\sqrt{\frac{1 + \cos 230^\circ}{2}} = ?$

II) Find the exact value for each expression.

1) $\cos 23^\circ \cos 37^\circ - \sin 23^\circ \sin 37^\circ$, 2) $\sin 54^\circ \cos 9^\circ - \cos 54^\circ \sin 9^\circ$, 3) $\frac{\tan(45^\circ) - \tan(75^\circ)}{1 + \tan(45^\circ) \tan(75^\circ)}$

4) $\cos \frac{2\pi}{9} \cos \frac{5\pi}{9} + \sin \frac{2\pi}{9} \sin \frac{5\pi}{9}$, 5) $\frac{\tan(-4^\circ) + \tan(-56^\circ)}{\tan(-4^\circ) \tan(-56^\circ) - 1}$, 6) $\frac{\tan(\pi) - \tan(\frac{9\pi}{12})}{1 + \tan(\pi) \tan(\frac{9\pi}{12})}$

III) Given $\sec \alpha = -3$, $\tan \alpha < 0$ and $\cot \beta = \sqrt{2}$, $\csc \beta < 0$ find the exact value for

a) $\sin(\alpha + \beta)$ b) $\tan(\alpha - \beta)$ c) $\cos(\alpha + \beta)$ d) $\tan \frac{\alpha}{2}$ e) $\sec \frac{\alpha}{2}$ f) $\sin \frac{\beta}{2}$ g) $\cos \frac{\beta}{2}$

h) $\cos 2\alpha$ i) $\sin 2\beta$ j) $\tan 2\alpha$ [note: $0 < \alpha, \beta < 2\pi$]

IV) Use the addition or subtraction formulas to find the exact value of $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the specified value of θ

1) $\theta = 105^\circ$ 2) $\theta = 15^\circ$ 3) $\theta = 165^\circ$ 4) $\theta = \frac{23\pi}{12}$ 5) $\theta = 285^\circ$ 6) $\theta = \frac{5\pi}{12}$ 7) $\theta = \frac{17\pi}{12}$

V) Use the addition or subtraction formulas to verify the identity.

1) $\cos(\theta + 30^\circ) + \cos(\theta - 30^\circ) = \sqrt{3} \cos \theta$ 2) $\cos(\theta + \frac{\pi}{6}) + \sin(\theta - \frac{\pi}{3}) = 0$ 3) $\frac{\sin(\theta + \frac{\pi}{2})}{\cos(\theta + \frac{\pi}{2})} = -\cot \theta$

4) $\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta$ 5) $\cos \alpha \cos(\alpha - \beta) + \sin \alpha \sin(\alpha - \beta) = \cos \beta$

VI) Use half angle formulas to find the exact value for the following

1) $\sin \frac{5\pi}{12}$ 2) $\tan 67.5^\circ$ 3) $\cos 112.5^\circ$ 4) $\sin \frac{7\pi}{8}$ 5) $\cot \frac{\pi}{12}$ 6) $\sin(-\frac{\pi}{8})$ 7) $\cos(-165^\circ)$

VII) Use the double angle formulas to find the exact value for the following

1) $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$ 2) $1 - 2 \sin^2 \frac{5\pi}{8}$ 3) $\sin 105^\circ \cos 105^\circ$ 4) $1 - 2 \cos^2 15^\circ$ 5) $\frac{\tan 22.5^\circ}{1 - \tan^2 22.5^\circ}$

VIII) If $\sin \theta = -\frac{\sqrt{3}}{3}$, $\frac{3\pi}{2} < \theta < 2\pi$ find

1) $\sin 2\theta$ 2) $\cos 2\theta$ 3) $\tan 2\theta$ 4) $\sin \frac{\theta}{2}$ 5) $\cos \frac{\theta}{2}$ 6) $\tan \frac{\theta}{2}$

IX) 1) Given $-2\pi < \theta < 0$. If $\sin \theta = -\frac{4}{5}$, and $\tan \theta > 0$ find $\cos \frac{\theta}{2}$

2) Given $-2\pi < \theta < 0$. If $\tan \theta = \frac{3}{4}$, and $\cos \theta > 0$ find $\sin \frac{\theta}{2}$

X) Use the double angle and the half angle formulas to verify the identity

1) $\frac{\sin \theta + \tan \theta}{2 \tan \theta} = \cos^2 \frac{\theta}{2}$, 2) $\frac{2}{\cot \theta - \tan \theta} = \tan 2\theta$, 3) $\frac{\sin 2\theta - \sin \theta}{\cos 2\theta + \cos \theta} = \tan \frac{\theta}{2}$

4) $\sin \theta \sec \frac{\theta}{2} = 2 \sin \frac{\theta}{2}$, 5) $\csc \theta = \cot \theta + \tan \frac{\theta}{2}$, 6) $\frac{\sec^2 \theta}{2 - \sec^2 \theta} = \sec 2\theta$

XI) 1) Find $\tan \theta$ if $\cos 2\theta = -\frac{1}{8}$, $\frac{\pi}{2} < \theta < \pi$

2) Find $\sec \theta$ if $\tan 2\theta = -\frac{8}{15}$, $\frac{3\pi}{2} < \theta < 2\pi$

XII) Evaluate the following using addition or subtraction or double angle or half angle

Formulas

1) $\cos \frac{\pi}{10} \cos \frac{2\pi}{5} + \sin \frac{\pi}{10} \sin \frac{2\pi}{5} =$

2) $\sin(-5^\circ) \cos(-7^\circ) - \sin(-7^\circ) \cos(-5^\circ) =$

3) $\frac{\tan(-75^\circ) + \tan(35^\circ)}{1 + \tan 75^\circ \tan 35^\circ} =$

4) $\sin(-10^\circ) \cos(-10^\circ) =$

5) $1 - 2 \sin^2 3\theta =$

6) $\sin^2 \frac{\theta}{6} - \cos^2 \frac{\theta}{6} =$

7) $2 \cos^2 \frac{\theta}{3} - 1 =$

8) $\sqrt{\frac{1 + \cos(-25^\circ)}{2}} =$

9) $\frac{\tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}} =$

10) $\frac{1 - \cos \frac{\pi}{11}}{\sin \frac{\pi}{11}} =$

11) $\frac{\sin \frac{7\pi}{5}}{1 + \cos \frac{7\pi}{5}} =$

12) $\sqrt{\frac{1 - \cos(-\frac{3\pi}{7})}{2}} =$

XIII) 1) If $\sin \theta = -\frac{3}{5}$ then $\cos(\frac{3\pi}{2} - \theta) =$

2) If $\sec \theta = -\frac{3}{2}$ then $\csc(\frac{3\pi}{2} + \theta) =$

3) If $\tan \theta = -3$ then $\cot(\frac{\pi}{2} + \theta) =$

4) If $\cot \theta = -\frac{1}{3}$ then $\tan(\frac{3\pi}{2} - \theta) =$

5) If $\csc \theta = -3$ then $\cos(\frac{\pi}{2} + \theta) =$

XIV) Find the exact value of each expression

1) $\tan[\frac{1}{2} \sin^{-1}(-1)]$, **2) $\cos(2 \cos^{-1} \frac{1}{2})$, **3) $\sin[\frac{1}{2} \tan^{-1}(-\sqrt{3})]$, **4) $\sin[2 \cos^{-1}(-\frac{\sqrt{3}}{2})]$******

5) $\tan[\frac{1}{2} \cos^{-1}(-1)]$, **6) $\cos(2 \sin^{-1} \frac{\sqrt{3}}{2})$, **7) $\sin[2 \cos^{-1}(-\frac{1}{\sqrt{2}})]$, **8) $\cos(2 \cos^{-1} 0)$, **9) $\tan[\frac{1}{2} \sin^{-1}(-\frac{4}{5})]$********

10) $\csc[2 \tan^{-1}(-\frac{3}{4})]$, **11) $\sec(2 \sin^{-1} \frac{3}{4})$, **12) $\sin(\frac{1}{2} \cos^{-1} \frac{4}{5})$, **13) $\cos[\frac{1}{2} \sin^{-1}(-\frac{12}{13})]$******

14) $\cos[\tan^{-1}(-1) + \cos^{-1}(-\frac{4}{5})]$, **15) $\tan[\sin^{-1}(-\frac{3}{5}) - \frac{\pi}{4}]$**