Abstract:
Let $K$ be a closed connected $k$-manifold, $0 \leq k \leq n - 1$. A subset $B$ in the $n$-manifold $M^n$ is $K$-contractible (in $M$) if there are maps $\varphi : B \to K$ and $\alpha : K \to M^n$ such that the inclusion map $i : B \to M^n$ is homotopic to $\alpha \cdot \varphi$. The $K$-category $\text{cat}_K M$ of $M$ is the smallest number of sets, open and $K$-contractible needed to cover $M$. For $K$ a point $P$ we obtain the classical Lusternik-Schnirelman category $\text{cat} M = \text{cat} P M$. We are interested here in the case $K = S^1$. We give some examples and discuss recent results.