

Section 5.1

Formulas you will need in this section:

Area of a sector:

$$A = \frac{1}{2}r^2\theta$$

Arc length:

$$s = r\theta$$

Angular velocity:

$$\omega = \theta/t$$

Linear velocity:

$$v = s/t \text{ or } v = r\omega$$

r = radius, θ = angle measure

For all formulas, θ must be in radians (including ω).

Angles

Initial side-

Terminal side-

Vertex-

Positive angle-

Negative angle-

Circles

Central angle-

Arc length-

Area of a sector-

Two units of angle measure are *degree* and *radian*.

A degree of a circle is $\frac{1}{360}$ of a revolution of the circle.

One radian is a length around the circle equal to the radius of the circle. In other words, suppose a central angle cuts an arc that is equal to the radius of the circle. Then that central angle has a radian measure of 1.

Let's take a look at a circle with a radius of 1 unit.

What is the radian measure of an angle equal to one complete revolution?

What is the radian measure for a straight angle (180°)?

What is the radian measure for an angle that is $1/4$ of the circle (90°)?

Conversions

Radians to degrees:

Example Convert 3 radians to degrees.

Degrees to radians:

Example Convert $\frac{120^\circ}{\pi}$ to radians.

Revolutions to radians:

Example How many radians are there in 3 revolutions?

Suppose you have a circle of radius r . And suppose a point is moving around that circle at a constant speed. We can look at two types of velocities:

- 1) How fast is the point moving about the circle (linear velocity)?
- 2) How fast is a central angle whose terminal side contains the point moving (angular velocity)?

Suppose we know how fast the angle is moving and we need to know the linear velocity:

Examples

- 1) A turntable with a radius of 10 feet is moving at a speed of 4 radians per minute. Find the linear speed (in feet per minute) of a point on the rim of the turntable.
- 2) Find a central angle (in radians) of a circle of radius 3 inches that subtends an arc of 6 inches.
- 3) Find the area of a sector of a circle with a central angle of 120° and radius 4 feet.
- 4) A windshield wiper is 30 inches long. How many inches will the tip of the wiper move during $\frac{1}{6}$ of a revolution?
- 5) An object is traveling in a circular path of radius 6 feet. If the object sweeps out an angle of 4 radians in 20 seconds, what is the angular speed of the object in radians per second?
- 6) A clock with a 3 meter pendulum moves from side to side through an angle of 0.4 radians. How many meters does the pendulum move from side to side?
- 7) Find the radius (in feet) of a circle if a central angle of 85° subtends a 200 foot arc.
- 8) A record of radius 10 inches is spinning at $33 \frac{1}{3}$ rpm. What is the linear speed, in in/min, of a point on the rim of the record?