6.3 Class Notes

Double angle formulas (note: each of these is easy to derive from the sum formulas letting both A=θ and B=θ)

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\qquad$ $\sin 2\theta = 2\cos\theta \sin\theta$ $\qquad$ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Half-angle formulas

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ $\qquad$ $\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ $\qquad$ $\tan\dfrac{\theta}{2} = \dfrac{\sin\theta}{1+\cos\theta}\ or\ \dfrac{1-\cos\theta}{\sin\theta}$

Let us derive the formula for the $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

$\cos 2\theta = \cos(\theta + \theta) = (\cos\theta)(\cos\theta) - (\sin\theta)(\sin\theta)$ $\qquad$ using the sum of two angles formula for cos (A + B)and letting A=B=θ.

$= \cos^2\theta - \sin^2\theta$

We can derive two more formulas for cos 2θ by manipulating the Pythagorean Identity: $\cos^2\theta + \sin^2\theta = 1$

Solve this for $\cos^2\theta$ and you have $\cos^2\theta = 1 - \sin^2\theta$. Now plug in to the double angle formula:

$\cos 2\theta = \cos^2\theta - \sin^2\theta = (1 - \sin^2\theta) - \sin^2\theta = 1 - 2\sin^2\theta$

Similarly, you can show that $\cos 2\theta = 2\cos^2\theta - 1$.

EXAMPLES:

1) Given $\cos\theta = \dfrac{2}{\sqrt{5}}$, $\dfrac{3\pi}{2} < \theta < 2\pi$, use a double angle formula to find sin 2θ.

2) Given $\sin \theta = -\frac{\sqrt{3}}{3}$, $\pi < \theta < \frac{3\pi}{2}$, use a double angle formula to find $\cos 2\theta$.

3) Given $\cos \theta = \frac{3}{\sqrt{10}}$, $0 < \theta < \frac{\pi}{2}$, use a double angle formula to find $\tan 2\theta$.

IDENTITIES

1) Select the formula for $\cos (x + y)$.

2) Select the formula for $\tan 2\theta$.

Half Angle Formulas

These can be tricky. You need to remember that the + or – in the formula depends upon the quadrant in which $\theta/2$ lies (not θ) along with the particular trig function you are evaluating.

EXAMPLES

1) Given $\cos\theta = -3/5$ and $\pi < \theta < 3\pi/2$, find the exact value of $\tan \theta/2$.

2) Use a half-angle formula to find the exact value of $\sin(21\pi/8)$.

Steps to solve these type problems:

1) Simplify the angle by subtracting off full rotations.
2) Determine the quadrant for $\theta/2$.
3) Double $\theta/2$. This is θ.
4) Find $\cos\theta$.
5) Plug in to the correct formula (look back to #2 to decide whether to use + or -)
6) Simplify. **Please note**: in these problems you will have trouble getting the correct answer unless you use rationalized numbers such as $\sqrt{2}/2$.

3) Use a half-angle formula to find the exact value of cos $(-29\pi/12)$.

4) Use a half-angle formula to find the exact value of sin $(-\pi/12)$.

4) Complete the following identity: $1 + \tan^2 \dfrac{\theta}{2}$

A) $-\csc^2 \dfrac{\theta}{2}$

B) $\sec^2 \dfrac{\theta}{2}$

C) $\csc^2 \dfrac{\theta}{2}$

D) $-\sec^2 \dfrac{\theta}{2}$