Section 8.5

DOT PRODUCT

For vectors $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$

$\mathbf{v} \bullet \mathbf{w} = a_1a_2 + b_1 b_2$ (a real number!!)

Example: $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$ and $\mathbf{w} = 5\mathbf{i} + 3\mathbf{j}$

Find $\mathbf{v} \bullet \mathbf{w}$.

Find $\mathbf{v} \bullet \mathbf{v}$.

Finding the angle, θ, between vectors **v** & **w**:

$$\cos\theta = \frac{v \bullet w}{\|v\| \cdot \|w\|} \qquad , 0 \le \theta \le \pi$$

Example: If $\mathbf{u} = 5\mathbf{i} - 6\mathbf{j}$ and $\mathbf{w} = -3\mathbf{i} - 5\mathbf{j}$, then *the cosine of the angle* between u and w is

Example: The angle, θ, between vectors $\mathbf{u} = \mathbf{i} - 7\mathbf{j}$ and $\mathbf{w} = 9\mathbf{i} - \mathbf{j}$ is:

a) $\cos^{-1}\left(\frac{8}{5\sqrt{41}}\right)$ b) $\pi/2$ c) $\cos^{-1}\left(\frac{1}{5\sqrt{41}}\right)$ d) $\cos^{-1}\left(\frac{8}{\sqrt{33}}\right)$

Example: The angle, θ, between vectors $\mathbf{u} = -7\mathbf{i} + 3\mathbf{j}$ and $\mathbf{w} = 7\mathbf{i} - 3\mathbf{j}$ is:

Parrallel, Orthogonal, or Neither

Example: Determine if $\mathbf{u} = \sqrt{3}\,\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = \mathbf{i} - \sqrt{3}\,\mathbf{j}$ are orthogonal, parallel, or neither.

Example: Determine if $\mathbf{u} = 3\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = \mathbf{i} + 3\mathbf{j}$ are orthogonal, parallel, or neither.

Example: If two vectors $\mathbf{u} = \frac{1}{4}\mathbf{i} - \frac{3}{5}m\mathbf{j}$ and $\mathbf{w} = \mathbf{i} + \frac{1}{3}\mathbf{j}$ are orthogonal then m =

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Pro**j**ect**i**on of v on w:

$\text{pro}\mathbf{j}_{\mathbf{w}}\mathbf{v} = \frac{v \bullet w}{\| w \|^2} w$

Example: The vector pro**j**ect**i**on of $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$ on $\mathbf{w} = 2\mathbf{i} - 2\mathbf{j}$ is:

Example: The vector pro**j**ect**i**on of $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$ orthogonal to $\mathbf{w} = 4\mathbf{i} - \mathbf{j}$ is:

EXTRA: Name a vector orthogonal to $2\mathbf{i} - \mathbf{j}$.

Work Problems

F = magn**i**tude of force x unit vector in the direction of the force
$\vec{AB}$ = vector in the direction of movement

w = work done by the force

w = F• $\vec{AB}$

Example: Find the work (in foot-pounds) done by a force of 3 pounds acting in the direction 2**i** + **j** in moving an object 2 feet from (0,0) to (0,2).