HOMEWORK 2 **DIFFERENTIAL TOPOLOGY** **MTG 5932-02**

due Friday, September 19

1. Let $f : R^2 \to R$ be a function of class C^1. Show that f is not injective. (Hint: The case in which $f'(x, y) = 0$ for all $(x, y) \in R^2$ is fairly easy (do it); next, suppose there is some $(x_0, y_0) \in R^2$ at which $f'(x_0, y_0) \neq 0$. Without loss of generality, assume the first partial derivative $f_x(x_0, y_0) \neq 0$. Then, consider the function $F : R^2 \to R^2$ defined by $F(x, y) = (f(x, y), y)$ and apply the Inverse Function Theorem. I'm not sure if this is how Lang intended the problem to be solved, but it should work!)

2. Let $f : R^2 \to R^2$ be defined by $f(x, y) = (e^x \cos y, e^x \sin y)$. Show that $f'(x, y)$ is invertible everywhere; hence conclude that f is locally invertible at all $(x, y) \in R^2$. But show that f does not have an inverse defined on all of $f(R^2)$.

3. Consider the manifold S^2 defined in class, and assume $(x, y, z) \in S^2$, with $z > 0$. Use the parameterization given in class (and in Milnor) to show that if $\vec{v} \in R^3$, then $\vec{v} \in TS^2_{(x,y,z)}$ if and only if $\vec{v} \cdot (x, y, z) = 0$.

4. Define $f : S^2 \to R$ by $f(x, y, z) = z$ for all $(x, y, z) \in S^2$. Calculate explicitly the map $df_{(0,0,1)} : TS^2_{(0,0,1)} \to R$ and show it is the zero map.

5. Let $F : C \to C$ be a complex analytic function. If you think of f as a map from $R^2 \to R^2$, it is smooth (why?). In terms of the complex derivative $f'(z_0)$ at a point $z_0 = x_0 + y_0 i$, what is the 2×2 matrix of $df_{(x_0,y_0)} : R^2 \to R^2$?