Topology II MTG5327 Spring 2014 Homework 5 Due Wednesday, March 5, 2014

Below I give you six problems: five from Hatcher and one extra one. Please hand in solutions to any five of these problems:

Five from Hatcher: Section 1.3, page 79-80: 3,4,9,10,14.

Here is the extra problem:

Suppose X is path connected, locally path connected, and semi-locally simply connected, and \tilde{X} is path connected, and $p:(\tilde{X}, \tilde{x}_0) \to (X, x_0)$ is a covering space map. Show that the following two conditions are equivalent:

(1) $p_*(\pi_1(\tilde{X}, \tilde{x}_0))$ is a normal subgroup of $\pi_1(X, x_0)$.

(2) Every loop $\gamma: I \to X$ has the following property: Either every lift of γ is a loop or no lift of γ is a loop. (If you want to assume that this property only holds for loops based at x_0 , you may do so; however the statement is true with or without this assumption.)