

Section 3.8 - Steps in Solving Polynomial and Rational Inequalities

STEP 1 : Write the inequality so that a polynomial or rational expression f is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0 \quad f(x) \geq 0 \quad f(x) < 0 \quad f(x) \leq 0$$

For rational expressions, be sure that the left side is written as a single quotient. **This step converts the problem of solving an inequality into an equivalent (i.e., the same solution) problem of determining where a function is positive (or negative).**

STEP 2 : Factor $f(x)$ to determine the numbers at which the expression $f(x)$ on the left side equals **zero** and, if the expression is rational, the numbers at which the expression f on the left side is **undefined**.

*We will call these numbers **partition values**.*

The **Intermediate Value Theorem** tells us that a continuous function (graph can be drawn without raising the pencil from the paper) **cannot change signs** on an interval without having a **zero** in that interval. So the above **partition values** divide the x -axis into intervals on which the sign of $f(x)$ **CANNOT** change.

STEP 3 : Use the numbers found in STEP 2 to separate the real number line into intervals.

We will construct a sign chart for the function $f(x)$.

STEP 4 : Determine the sign of $f(x)$ on the intervals found in step 3.

STEP 5: The solution of the inequality includes all intervals with the correct sign (positive or negative).

If the inequality is not strict, include the numbers at which $f(x)$ is zero in the solution set.

Be careful: *The numbers which make $f(x)$ undefined (i.e., the zeroes of the denominator in a rational function) are never included in the solution set.*