

Review of Exponents and Radicals: (see page 967-970 in the text)

<u>Rules and definitions (x,y real numbers; m,n integers)</u>		<u>examples</u>	
1. $x^0 = 1$	definition, for $x \neq 0$	$7^0 = 1$	$e^0 = 1$
2. $x^{-n} = \frac{1}{x^n}$	definition of negative exponents, $x \neq 0$	$5^{-2} = \frac{1}{5^2}$	$e^{-3} = \frac{1}{e^3}$
3. $x^m x^n = x^{m+n}$	product rule	$2^3 2^5 = 2^8$	$e^4 e^3 = e^7$
4. $(x^m)^n = x^{mn}$	power of a power rule	$(3^2)^5 = 3^{10}$	$(e^3)^4 = e^{12}$
5. $\frac{x^m}{x^n} = x^{m-n}$	quotient rule	$\frac{2^7}{2^2} = 2^5$	$\frac{e^6}{e^2} = e^4$
6. $(xy)^n = x^n y^n$	power of a product rule	$3^2 3^5 = 3^7$	$e^3 e^4 = e^7$
7. $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	power of a quotient	$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$	$\left(\frac{e}{3}\right)^2 = \frac{e^2}{3^2}$
8. $x^{\frac{1}{n}} = \sqrt[n]{x}$	def., ( $n \geq 2$ ) and ( $x > 0$ or $n$ is odd)	$(32)^{\frac{1}{5}} = \sqrt[5]{32} = 2$	$e^{\frac{1}{3}} = \sqrt[3]{e}$
9. $x^{\frac{m}{n}} = \sqrt[n]{x^m}$	def. - ( $n \geq 2$ ) and ( $x > 0$ or $n$ odd)	$(32)^{\frac{2}{5}} = (\sqrt[5]{32})^2 = 2^2 = 4$	

Notice that if  $x = -1$  and  $n = 2$ , then the expression  $x^{\frac{1}{n}} = (-1)^{\frac{1}{2}} = \sqrt{-1}$  is not defined in the set of real numbers (since the square root of a negative number does not exist in the set of reals.)

However, if  $b$  is a positive real number, then the expression  $b^n$  can be defined for any real number  $n$ .

(See the discussion on page 294 of your text. Try evaluating  $2^{\sqrt{2}}$  or  $2^{-\pi}$  on your calculator.)

When defining the exponential function,  $f(x) = b^x$ , the base  $b$ , is required to be positive so that  $f$  is defined for any real  $x$ , i.e., so the domain of  $f$  is  $(-\infty, \infty)$ .