10.4 - **Cramer's Rule -** The solution to the system of equations:

$$a_{11}x + a_{12}y + a_{13}z = c_1$$
$$a_{21}x + a_{22}y + a_{23}z = c_2$$
$$a_{31}x + a_{32}y + a_{33}z = c_3$$

is given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, $z = \frac{D_z}{D}$ where

where D = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, $D_x = \begin{vmatrix} c_1 & a_{12} & a_{13} \\ c_2 & a_{22} & a_{23} \\ c_3 & a_{32} & a_{33} \end{vmatrix}$, $D_y = \begin{vmatrix} a_{11} & c_1 & a_{13} \\ a_{21} & c_2 & a_{23} \\ a_{31} & c_3 & a_{33} \end{vmatrix}$, $D_z = \begin{vmatrix} a_{11} & a_{12} & c_1 \\ a_{21} & a_{22} & c_2 \\ a_{31} & a_{32} & c_3 \end{vmatrix}$.

---

**More on Matrices:**

Be able to solve systems of equations by (1) applying elementary row operations to the augmented matrix followed by back substitution and (2) using the inverse of the coefficient matrix.

Be able to add matrices, multiply a matrix by a scalar, multiply a matrix by another matrix, calculate the determinant, find the inverse and put the matrix in echelon or reduced row echelon form.

---

# Arithmetic Sequences

A sequence of numbers $a_1, a_2, a_3, \cdots, a_n, \cdots$

is called an **Arithmetic Sequence**

if there is a constant ***d***, called the **<u>common difference</u>**, such that

$$a_n = a_{n-1} + d, \quad \text{for every } n > 1.$$

Or, solving for ***d***: $d = a_n - a_{n-1}$

---

The **nth term of an Arithmetic Sequence** is:

$a_n = a_1 + (n-1)d$, with $a_1$=first term; d=common difference.

The **sum of the first *n* terms** of an Arithmetic Sequence is:

$$S_n = \frac{n}{2}(a_1 + a_n)$$ or, in alternate form,

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

---

# Geometric Sequences

A sequence of numbers $a_1, a_2, a_3, \cdots, a_n, \cdots$

is called an **Geometric Sequence**

if there is a constant **r**, called the **common ratio**, such that

$\mathbf{a_n = ra_{n-1}}$ , for every $n > 1$, $r \neq 0$.

Solving for **r**: $r = \frac{a_n}{a_{n-1}}$

---

The **nth Term of a Geometric Sequence** is:

$\mathbf{a_n = a_1 r^{n-1}}$ , where $a_1$=first term; r=common ratio.

---

The **Sum of an Infinite Geometric Series**

If |r|<1, the sum of the infinite geometric series is:

$$S = \sum_{k=1}^{\infty} ar^{k-1}$$

is:

$$S = \frac{a}{1-r}$$ , where a = first term, r = common ratio.